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A Weighted Fractional Order PID Controller for Nonlinear Systems with Variable Delay ⋆

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Abstract: This paper presents a Weighted Fractional Order PID (WFO-PID) controller that enhances the conventional Fractional Order PID (FOPID) structure by introducing explicit weighting factors for the tuning terms. These weights, combined with fractional orders and gain parameters, provide greater flexibility in tuning, enabling for more precise control over system dynamics. The proposed controller is applied to a mixing tank process with variable time delay, a representative case of industrial processes with challenging dynamics. Its performance is benchmarked against both PID and FOPID controllers, demonstrating improved reference tracking and disturbance rejection, particularly under varying delay conditions and noise in the transmitter. The findings highlight the WFO-PID method's efficiency in managing intricate control situations.

Keywords: Weighted Operators, Fractional systems, Fractional PID control, Composition properties, Industrial Process Control.

1. INTRODUCTION

Control systems play a critical role in industrial processes by regulating performance around a specified set point. Achieving optimal operation requires continuous adjustment of system components and parameters within acceptable bounds. Among various industrial sectors, the chemical industry holds a significant global presence due to its central role in producing essential goods, including pharmaceuticals, food, and petrochemical products. Given the highly nonlinear and dynamic nature of chemical processes, robust and efficient control strategies are essential to ensure stability and satisfactory performance. In this context, regulating system parameters becomes a key challenge, as it must account for the complex and time-varying behavior of the manipulated variables.

Proportional–Integral–Derivative (PID) controllers remain one of the most widely used control strategies in industrial and engineering applications due to their simplicity, robustness, and effectiveness in a wide range of systems (Dubey et al. (2022)). However, conventional PID controllers may exhibit limitations when applied to processes with complex dynamics, such as those with long memory effects, time delays, or nonlinear behavior. To overcome these limitations, Fractional Order PID (FOPID) controllers, which extend the traditional PID by introducing non-integer orders of integration and differentiation, have been proposed as a powerful generalization (Podlubny (1999b); Bingi et al. (2020)).

The FOPID controller, often represented as $PI^{\lambda}D^{\mu}$, offers two additional tuning parameters, λ and μ, which provide greater flexibility in shaping the system's time and frequency responses. These fractional orders enable more precise control over dynamic behavior, particularly in systems that exhibit viscoelasticity, diffusion, or hereditary characteristics, where conventional integer-order controllers may fall short (Yokuş et al. (2024)).

Despite their advantages, fractional-order controllers may still require further customization to address performance trade-offs across different operational conditions (Gude et al. (2024)). In particular, weighting the fractional terms or their gains provides an opportunity to balance performance criteria such as robustness, disturbance rejection, tracking accuracy, and energy efficiency (Ilten (2023); Bingi et al. (2018)).

This paper introduces a Weighted Fractional Order PID (WFO-PID) controller, which extends the traditional FOPID structure by adding explicit weighting factors. These weights, along with the fractional orders and gain parameters, offer greater flexibility in the tuning process, enabling more accurate control of system dynamics. The controller is tested on a mixing tank process with variable time delay—a typical example of industrial systems with challenging behavior. Its performance is compared with conventional PID and FOPID controllers, demonstrating

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improved reference tracking and disturbance rejection, particularly in the presence of measurement noise. The results confirm the effectiveness of the WFO-PID controller as an alternative to control industrial processes.

The paper is organized as follows. Section 2 reviews the fundamentals of fractional calculus, and Section 3 introduces the fractional composition operators. Section 4 presents a new fractional operator, while Section 5 describes its weighted formulation and numerical implementation. Section 6 details the criteria for selecting weighting functions. Section 7 discusses the results, and Section 8 concludes with the main contributions and future research directions.

2. FRACTIONAL CALCULUS

Definition 1. Let $\left(D_a^{\mu} f\right)(t)$ denote the **fractional Riemann Liouville derivative** of order $\mu > 0$ (see Kilbas et al. (2006); Podlubny (1999a); Samko et al. (1993)) and let it be defined as:

$$\left(D_a^{\mu} f\right)(t) = \left(\frac{d}{dt}\right)^p \left(I_a^{s-\mu} f\right)(t) = \left(\frac{d}{dt}\right)^p \frac{1}{\Gamma(p-\mu)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-p-\mu}}\, d\tau, \quad (1)$$

where $p = [\mu] + 1$, $t > a$, $[\mu]$ denotes the integer part of μ and Γ is the gamma function.

Remark 1. The function $f \in L_1(a, b)$ is said to have a summable fractional derivative $\left(D_a^{\mu} f\right)(t)$ if $\left(I_a^{p-\mu} f\right)(t) \in AC^p([a, b])$, where $p = [\mu] + 1$ and $AC^p([a, b])$ denotes the class of functions f whose $(p-1)$-th derivative is absolutely continuous on $[a, b]$.

When $0 < \mu < 1$, expression (1) simplifies to: $\left(D_a^{\mu} f\right)(t) = \frac{d}{dt}\left(I_a^{1-\mu} f\right)(t)$. Observe that as $\mu \to 1$, the standard derivative operator is recovered (Kilbas et al. (2006); Podlubny (1999a); Samko et al. (1993)). The semigroup property for the composition of fractional derivatives does not hold generally (see (Podlubny, 1999a, Sect. 2.3.6)). In fact, the property:

$$D_a^{\mu}\left(D_a^{\beta} f\right) = D_a^{\mu+\beta} f, \quad (2)$$

holds if

$$f^{(j)}(a) = 0, \qquad j = 0, 1, \ldots, s-1, \quad (3)$$

and $f \in AC^{s-1}([a, b])$, $f^{(s)} \in L_1(a, b)$ and $s = [\beta] + 1$.

Example 1. Let $\mu, \lambda \in (0,1)$, $a > 0$, k $\in \mathbb{N}$ and $\beta > -1$, then

(1) $I_a^{\lambda}\left[(t-a)^{\beta}\right] = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\lambda+1)}(t-a)^{\beta+\lambda}$

(2) $D_a^{\mu}\left[(t-a)^{\beta}\right] = \begin{cases} 0, & \beta = \mu - 1, \\ \dfrac{\Gamma(\beta+1)}{\Gamma(\beta-\mu+1)}(t-a)^{\beta-\mu}, & \text{otherwise.} \end{cases}$

Remark 1. It is worth noticing that the **Riemann-Liouville derivative of a constant is not zero**. However, in the limit process, it recovers the expected behavior.

$$\lim_{\mu \to 1}\left(D_a^{\mu} 1\right)(t) = \lim_{\mu \to 1} \frac{(t-a)^{-\mu}}{\Gamma(1-\mu)} = 0. \quad (4)$$

Remark 2. There are derivatives such as the Caputo derivative, in which the derivative of a constant is zero. See Kilbas et al. (2006); Podlubny (1999a); Samko et al. (1993)

3. FRACTIONAL COMPOSITION OPERATORS

This section aims to show how fractional operators are compositions of differential and integral operators. We explore various operators that lead to the central operator studied.

Let Θ_d be a differential operator and Θ_i an integral operator. Consider the general composition: $Comp = \Theta_d \circ \Theta_i \circ \cdots \circ \Theta_d \circ \Theta_i$

As a first example, we examine two compositions where $\mu \in (0, 1)$, Θ_d is the ordinary differential operator and Θ_i is the fractional integral of order μ

$$Comp_{[\mu]}(\cdot) = \frac{d}{dt} \circ I_a^{1-\mu} \circ \frac{d}{dt} = \frac{d}{dt}\left(I_a^{1-\mu} \frac{d}{dt}\right)(\cdot)$$

This expression can be interpreted as the derivative of a Caputo derivative, or equivalently, the Riemann–Liouville derivative of a first derivative. Another possibility: Consider $\mu \in (0, 1)$, and let $m, n \in \{0, 1\}$, where we define $\frac{d^0}{dt^0} f(t) = f(t)$. Then:

$$Comp_{[n,m,\mu]}(f)[t] = \frac{d^n}{dt^n} \circ I_a^{1-\mu} \circ \frac{d^m}{dt^m} f(t)$$

If $n = 1$ and $m = 0$, we obtain:

$$\begin{aligned} Comp_{[n=1,m=0,\mu]}(f)[t] &= \frac{d}{dt} \circ I_a^{1-\mu} \circ \frac{d^0}{dt^0} f(t) \\ &= \frac{d}{dt} I_a^{1-\mu} f(t) = D_a^{\mu} f(t) \quad (5) \end{aligned}$$

This is the Riemann–Liouville derivative of order μ. Conversely, if $n = 0$ and $m = 1$, we get:

$$\begin{aligned} Comp_{[n=0,m=1,\mu]}(f)[t] &= \frac{d^0}{dt^0} \circ I_a^{1-\mu} \circ \frac{d}{dt} f(t) \quad (6) \\ &= I_a^{1-\mu} \frac{d}{dt} f(t) = {}_C D_a^{\mu} f(t) \quad (7) \end{aligned}$$

That is, the Caputo derivative of order μ. Based on this idea of composition, we can motivate the weighted q-operator, introduced in Contreras et al. (2025), which has been applied to problems in physics and inspired by works such as Sousa and De Oliveira (2018), Fernandez and Fahad (2022), and Katugampola (2011). An important feature of this operator is that the derivative of a constant is zero for a certain class of functions, which facilitates its application to initial value problems.

Ongoing research aims to establish rigorous mathematical results that enable broader practical use of this operator. In this work, we focus on its numerical implementation via the Grünwald–Letnikov approach, which provides a direct discretization of fractional derivatives for use in simulations where analytical solutions are intractable.

Finally, since we work in the s-domain, it is important to consider that applying the Laplace transform results in convolutions in the t-domain, a topic that will be addressed in detail in the next section.

4. THE OPERATOR ${}^{\overline{q}}D_a^{\mu}$

Definition 2. Consider $q_1(t, \mu)$ as a continuous function and $q_2(t, \mu)$ as a continuously differentiable function with respect to t, and let $\left({}^{\overline{q}}D_a^{\mu} f\right)(t) = \left({}^{(q_1,q_2)}D_a^{\mu} f\right)(t)$ denote the **$\overline{q}$-weighted fractional Riemann Liouville derivative** of order $\mu > 0$. Assume that $q_1, q_2 \in AC^s(\mathbb{R})$. Then,

$$\left({}^{\overline{q}}D_a^\mu f\right)(t) = \left(q_1(t,\mu)\frac{d}{dt}\right)^s q_2(t,\mu)\left(I_a^{s-\mu} f\right)(t), \quad (8)$$

where $s = [\mu] + 1, t > a$, and $[\mu]$ denotes the integer part of μ.

If $0 < \mu < 1$ and $\lim_{\mu \to 1} q_1(t,\mu) = \lim_{\mu \to 1} q_2(t,\mu) = 1$, then we recover the classical case.

Example 2. For convenience, if we take $q_2(t,\mu) = (t-a)^{\mu-1}$, then (8) takes the form for $0 < \mu < 1$.

$$\left({}^{\overline{q}}D_a^\mu f\right)(t) = q_1(t,\mu)\left(\frac{d}{dt}\right)(t-a)^{\mu-1}\left(I_a^{1-\mu} f\right)(t). \quad (9)$$

and $\left({}^{\overline{q}}D_a^\mu 1\right)(t) = 0$.

4.1 Some properties of the $\overline{q}$-weighted operator

It is straightforward to verify that the operator is linear. As with the Riemann–Liouville operator, the semigroup property and the Leibniz rule do not generally hold for the weighted fractional operator. The following lemma presents a property analogous to the semigroup property, combining the $\overline{q}$-weighted fractional operator with the Riemann–Liouville derivative.

Lemma 3. Let $\mu, \beta > 0$ with $\mu + \beta < 1$, and suposse that $f(t) \in I_a^\beta(L_1)$. Then, the following property holds almost everywhere on $[a,b]$:

$$ {}^{\overline{q}}D_a^\mu(D_a^\beta f) = {}^{\overline{q}}D_a^{\mu+\beta} f. \quad (10)$$

Proof 1. Let μ, β be positive numbers such that $\mu+\beta < 1$, and $f(t) \in I_a^\beta(L_1)$. Then, using Lemmas 2.3 and 2.5 from Kilbas et al. (2006), and recalling the definition of the weighted fractional derivative operator given in equation (8), we have:

$$\begin{aligned} {}^{\overline{q}}D_a^\mu(D_a^\beta f) &= \left(q_1\frac{d}{dt}\right)^s \left(q_2\left(I_a^{s-\mu}D_a^\beta f\right)\right) \\ &= \left(q_1\frac{d}{dt}\right)^s \left(q_2\left(I_a^{s-\mu-\beta}I_a^\beta D_a^\beta f\right)\right) \\ &= \left(q_1\frac{d}{dt}\right)^s \left(q_2\left(I_a^{s-\mu-\beta} f\right)\right) = {}^{\overline{q}}D_a^{\mu+\beta} f. \end{aligned}$$

where $s = \lceil \mu \rceil$. This expression follows directly from applying the operator definition in (8) to the function $D_a^\beta f$.

5. THE $\overline{q}$-WEIGHTED OPERATOR WITH CONVOLUTIONS AND ITS NUMERICAL IMPLEMENTATION USING GRÜNWALD–LETNIKOV

Based on the operator defined in (8) when $0 < \mu < 1$, and considering numerical implementations with increased simplicity and computational efficiency, we slightly modify the structure of the operator, preserving its form, but now interpreting it in terms of convolutions.

The Grünwald-Letnikov derivative is defined as Palacios et al. (2023); Podlubny (1999a):

Definition 4. Let $\mu > 0$, $f \in C^k[a,b]$, and $a < x \le b$. Then $\mathcal{G}_a^\mu f(t) = \lim_{N\to\infty} \frac{\Delta_{h_N}^\mu f(t)}{h_N^\mu} = \lim_{h\to 0}\frac{1}{h^\beta}\sum_{k=0}^{\infty}(-1)^k\binom{\beta}{k} f(t-kh)$, with $h = \frac{(t-a)}{N}$, $N = 1, 2, \ldots$.

This definition involves a sum over shifted values of the function $f(t)$ and is suitable for discrete approximations.

Remark 5. It is worth mentioning that the Grünwald-Letnikov derivative can be seen as a discrete approximation of the Riemann-Liouville derivative. In the limit as the step size goes to zero, the Grünwald-Letnikov derivative converges to a continuous fractional derivative.

Having defined the Grünwald-Letnikov, we now define our version of the $\overline{q}$-weighted operator, which is employed for numerical implementations in this work:

$$\begin{aligned} \left({}^{\overline{q}}\mathcal{G}_x^\mu h\right)(t) &= \\ q_1(t,\mu) * \frac{d}{dt} q_2(t,\mu) * \left(I_a^{1-\mu} h\right)(t) &+ \\ q_1(t,\mu) * q_2(t,\mu) * \left(\mathcal{G}_x^\mu h\right)(t) & \quad (11) \end{aligned}$$

We are aware that using the properties of convolution with respect to differentiation could significantly simplify the notation in operator (11). However, we have chosen to present it this form to facilitate the analysis of the individual behavior of the function and the weights by expressing it as a linear combination.

6. CRITERIA FOR SELECTING THE WEIGHTS

There are various ways to establish criteria for the weights. These criteria depend on the convergence of the mathematical objects and on the model itself. Undoubtedly, the problem of defining appropriate criteria for the weights is neither unique nor simple to solve. In this paper, the criterion we adopt is a combination of ideas drawn from inverse problems, stability, speed, and other considerations.

The procedure consists of establishing a pseudocode with some assumptions that, from the implementation perspective, allow identifying when to exit the algorithm and, from there, assess the consistency of the mathematical objects

(1) First Ansatz: $q_1 \in \mathbb{R}^+$
We fix this weight for simplicity. Additionally, q_1 is linked to the optimization of a cost function involving both ISE and ISCO.
(2) Second Ansatz: q_2
is a real-valued function that governs the rate of convergence to the reference signal.

Consider $q_1, q_2 : \Omega \subseteq \mathbb{R} \to \Omega \subseteq \mathbb{R}$ such that $q_2 \in L^1(\Omega)$, and assume the first Ansatz. The operator is then defined:

$$\begin{aligned} \left({}^{\overline{q}}D_a^\mu f\right)(t) &= \\ q_1\frac{d}{dt}q_2(t,\mu) * \left(I_{a^+}^{s-\mu}h\right)(t) &+ q_1 q_2(t,\mu) * D_a^\mu f(t) \end{aligned}$$

For the value of q_1, we compute the cost function given by $cf := a \cdot ISE + b \cdot ISCO$, where $a, b \in \mathbb{R}$. Based on this, q_1 is defined by $q_1 = \min_{\xi}(cf)$, with ξ representing the fifty time steps considered, aiming at system stabilization.

Finally, a necessary condition for existence of the integral is that $q_2 \in C^1(\Omega)$ and $D_a^\mu f(t)$ and $(I_a^{s-\mu} f)(t)$ must be locally integrable on $[0, \infty)$ and bounded.

This requires that q_2 be of exponential type: $|q_2(t,\mu)| \leq Ae^{B\mu}$, where $t \in \Omega \subset \mathbb{R}$ and $A, B \in \mathbb{R}$.

Example 3. Let us consider the case where the weights are defined as $q_1 = k$ ($k \in \mathbb{R}$) and $q_2 = \sinh(t)$. The $\overline{q}$-weighted operator in this scenario, using the Grünwald–Letnikov approximation for $\mu \in (0, 1)$, becomes:

$$\left({}^{\overline{q}}\mathcal{G}_t^\mu f\right)(t) = k\,\cosh(t) * \left(I_a^{1-\mu} f\right)(t) + k\,\sinh(t) * \mathcal{G}_t^\mu f(t)$$

7. RESULTS

7.1 Mixing Tank

This model describes a mixing tank with hot and cold water inlet streams, where the temperature is measured by a sensor located (L = 125) ft downstream, as shown schematically in Fig. 1. The nonlinear dynamics, including variable dead time and nominal operating conditions, are detailed in Camacho and Smith (2000); Vásquez et al. (2023).

To develop a simplified model of the system, the reaction curve method described in Smith and Corripio (2005) was applied. This procedure yielded a First-Order Plus Dead-Time (FOPDT) model:

$$G_{p_1}(s) = \frac{-0.8215}{2.3s + 1} e^{-3.97s} \quad (12)$$

This result indicates that the process dynamics are dominated by time delay.

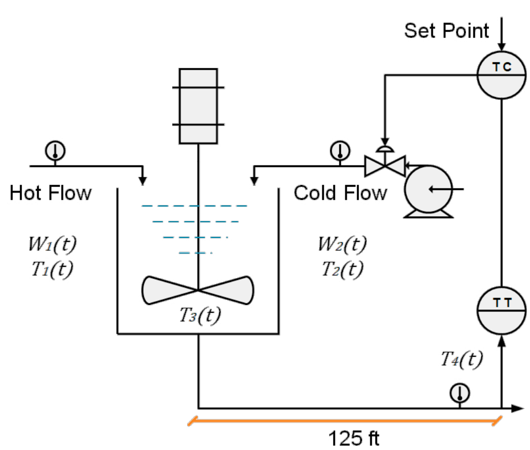

Fig. 1. Mixing Tank process.

7.2 Tuning Controller

Controller parameters were tuned using Dahlin's method Smith and Corripio (2005), which provides a systematic way to optimize the gains of the PID, FO-PID, and WFO-PID controllers. This approach enhances system performance by minimizing error, maintaining low steady-state deviations, and improving noise rejection across different operating conditions. Additionally, the value of μ was determined using particle swarm optimization (PSO), starting with an initial value in the range $[0, 1]$, to obtain a controller that lies between a PI and a PID controller. The final gains were: $K_P = 0.3526$, $T_I = \tau$, $T_D = \tau$ and $\mu = 0.52$

7.3 Reference Changes

Figure 2 shows the transmitter output in response to a step change in the reference. The WFO-PID delivers the best performance, with fast stabilization and moderate overshoot. The FO-PID offers a more balanced response than the classical PID, reducing overshoot and oscillations. In contrast, the PID exhibits the largest overshoot and the slowest settling, making it less suitable for sensitive applications.

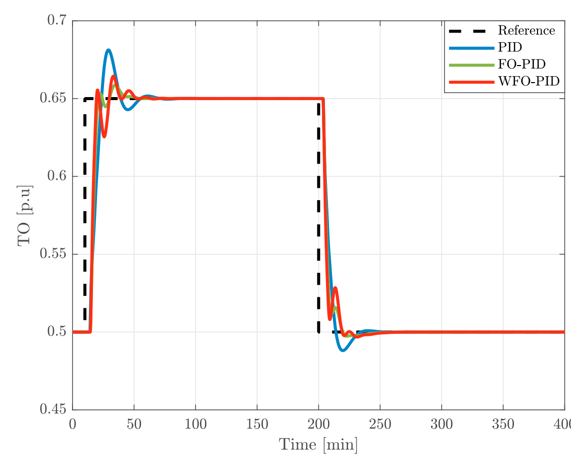

Fig. 2. Output of the mixing tank under reference changes.

Figure 3 shows the control signals for the three controllers. The PID produces a smooth output but responds slowly to setpoint changes. The FO-PID reacts faster and rejects disturbances better. The WFO-PID exhibits a brief negative peak near 200 minutes due to the derivative action on the abruptly changing variable q_2, but quickly restores stable control. This illustrates the sensitivity of fractional-order designs to sudden transients, which can be reduced through proper tuning and filtering.

Table 1 summarizes the controllers' performance during the final reference changes. All controllers track the setpoint, but the WFO-PID achieves the lowest error and overshoot with minimal effort. The FO-PID also improves significantly over the classical PID, offering faster settling and reduced actuation. In contrast, the PID shows the highest overshoot and control effort, indicating a more aggressive and less efficient response. These results highlight the superior adaptability and efficiency of fractional-order approaches in response to dynamic setpoint variations.

Table 1. Comparison of Performance parameters for the last reference changes

Controllers	**ISE**	**ISCO**	**t_s [min]**	**M_p [%]**
PID	0.28	130.80	3.00	280
FO-PID	0.26	125.30	1.50	277
WFO-PID	0.25	125.30	1.50	270

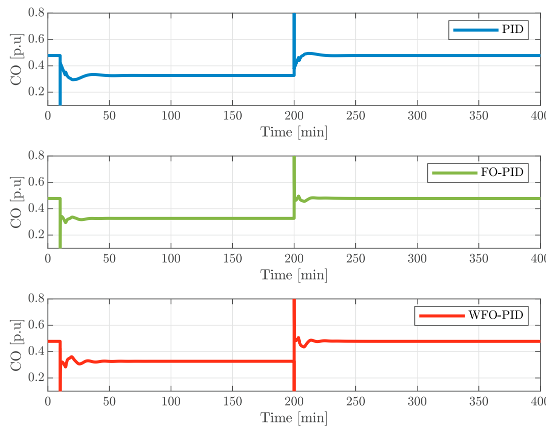

Fig. 3. Controller Output of the mixing tank under reference changes.

7.4 *Regulation*

Figure 4 shows the TO system's response to hot water flow rate disturbances W_1 in a mixing tank, comparing PID, FO-PID, and WFO-PID controllers against the desired temperature. The disturbances cause deviations in the output temperature. The classical PID exhibits unstable behavior with large, sustained oscillations, indicating poor disturbance rejection. The FO-PID improves performance with reduced oscillations and faster recovery. The WFO-PID outperforms all others, rapidly suppressing disturbances with minimal overshoot and excellent stability, closely maintaining the desired temperature throughout.

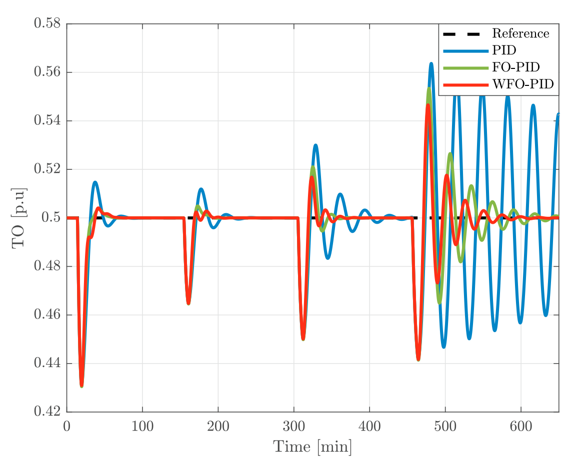

Fig. 4. Transmitter output of the mixing tank under hot water disturbances.

Figure 5 shows the control signals in response to disturbances in the hot water flow rate W_1. The classical PID exhibits strong oscillations, particularly after 450 minutes, indicating overly aggressive behavior. The FO-PID provides a smoother response with fewer oscillations and better disturbance rejection. The WFO-PID achieves the most stable performance, producing a step-like control action with minimal effort. Overall, the WFO-PID offers the best compromise between temperature regulation and control efficiency.

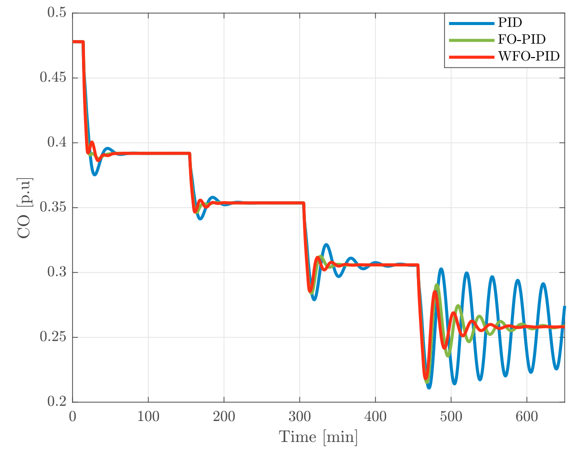

Fig. 5. Controller Output of the mixing tank under hot water disturbances.

Table 2 compares controller performance during the final disturbance event. The WFO-PID achieves the best results, with the lowest error, minimal effort, and the fastest settling, while keeping overshoot small. The FO-PID improves upon the classical PID across all metrics, though with lower efficiency than the WFO-PID. The PID shows the weakest response, with higher error, larger overshoot, and no convergence within the evaluated time. These results confirm the superior disturbance handling and efficiency of the WFO-PID.

Table 2. Comparison of Performance parameters for the last disturbance

Controllers	**ISE**	**ISCO**	**t$_s$ [min]**	**M$_P$ [%]**
PID	0.332	70.81	–	12.73
FO-PID	0.113	70.79	640.00	10.68
WFO-PID	0.100	70.78	570	9.32

7.5 *Noise*

Figure 6 presents the transmitter output under disturbances and measurement noise. The overall behavior is similar to the noise-free case, though the PID exhibits stronger oscillations, especially toward the end. The FO-PID maintains moderate improvements, while the WFO-PID delivers the most stable and accurate tracking. These results confirm the robustness of the WFO-PID in realistic, noisy operating conditions.

Figure 7 shows the control action under disturbance and noise. The PID remains saturated for most of the test, showing poor noise rejection and adaptability. The FO-PID provides a smoother, bounded response with improved efficiency. The WFO-PID performs best, maintaining low control effort and responding only to disturbances, thus ensuring stability even under noisy conditions.

Table 3 compares controller performance under the last disturbance combined with white noise. The WFO-PID controller maintains the lowest tracking error and control effort, showcasing its robustness in noisy environments. The FO-PID shows a clear improvement over the PID, which exhibits the highest error and control effort, highlighting its reduced effectiveness in the presence of

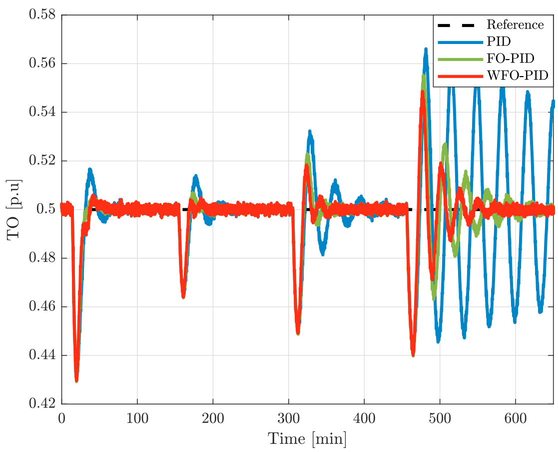

Fig. 6. Transmitter output of the mixing tank under hot water disturbances with white noise.

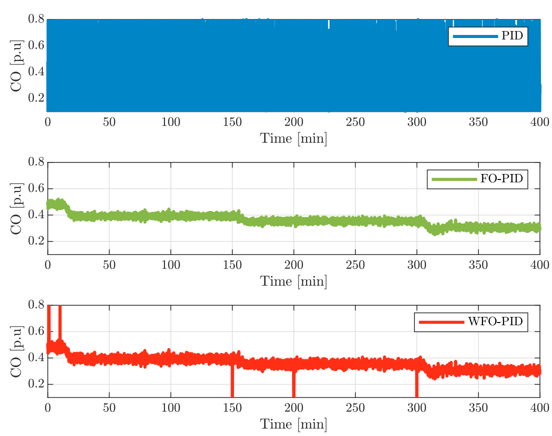

Fig. 7. Controller Output of the mixing tank under hot water disturbances with white noise.

measurement noise. These results reinforce the benefits of fractional-order and weighted fractional-order control strategies in noisy conditions.

Table 3. Comparison of Performance for the last disturbance with white noise

Controllers	**ISE**	**ISCO**
PID	0.34	72.42
FO-PID	0.12	70.8
WFO-PID	0.10	70.8

8. CONCLUSIONS

This work proposed a Weighted Fractional Order PID controller that improves the flexibility and performance of conventional FO-PID designs by introducing explicit weighting factors in the tuning process. The method was tested in a mixing tank with variable delay, demonstrating superior reference tracking and disturbance rejection compared to PID and FO-PID, even in the presence of measurement noise. These results highlight the WFO-PID as a robust option for complex industrial systems. Future work will address multivariable applications and real-time implementation.

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