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Extractive institutions and the takeoff to long-run growth: A Schumpeterian perspective

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ABSTRACT

We examine how extractive institutions affect the timing of the takeoff to sustained economic growth, the pace of industrialization, and the long-run balanced growth path of an economy. The politically dominant ruling elite can choose to extract a share of output and/or to interfere with creative destruction by extracting innovation resources. In so doing, the ruling elite needs to balance its desire for grabbing a greater share of resources with the constraint of being able to stay in power. We show that extraction from output delays the takeoff to sustained economic growth and reduces economic growth in the early industrial period. However, taken by itself, output extraction does not reduce the long-run balanced growth rate. By contrast, if the ruling elite interferes with creative destruction by extracting resources meant for innovation, it suppresses economic growth during industrialization and along the balanced growth path. After deriving the main results analytically, we calibrate the model to the U.S. economy to illustrate the adverse long-run development effects of extractive institutions. According to our results, institutions and policies that reduce the extractive power of the ruling elite can boost economic development to a substantial degree.

1. Introduction

The takeoff to sustained long-run economic growth marked a historic shift away from Malthusian stagnation, an era during which virtually all of the world's population lived close to the subsistence level (Malthus, 1798; Maddison Project Database, 2023). This takeoff was the precondition for economic development and enabled the unprecedented levels of prosperity we observe today in high-income countries. Many different forces have caused the takeoff, with the most important one being described in Unified Growth Theory (Galor and Weil, 2000; Galor and Moav, 2004, 2006; Galor, 2005, 2011) as the reversal of the positive association between income and fertility through the increasing importance of education in a technologically advancing world. Households started to invest in the education of their children and, thus, had to reduce the number of children because of limited household resources. This led to a demographic transition from high fertility to low fertility, which eventually allowed income to outgrow population and economies to escape the Malthusian stagnation.

Many complementary mechanisms reinforced the dynamics of the takeoff to sustained growth such as women's empowerment, which led to a boost in labor supply and also contributed to the fertility decline (Galor and Weil, 1996; Lagerlöf, 2003; Prettner and Strulik, 2017); falling child mortality, which dampened the precautionary fertility motive (Kalemli-Ozcan, 2003); falling adult mortality, which raised incentives to invest in human capital (Ben-Porath, 1967; Cervellati and Sunde, 2005, 2013, 2015); structural

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change driven by increasing productivity in agriculture, which led to a rise in the workforce available for manufacturing (Kögel and Prskawetz, 2001); and reductions in the political power of landowners who suppressed support for efficient public education policies (Galor et al., 2009).

However, while Unified Growth Theory successfully explains the timing of the takeoff to long-run economic growth through demographic and educational channels, it remains puzzling why economies with a similar timing of their demographic transitions could nevertheless experience a very different timing in terms of their economic takeoffs. For example, England and Spain both had the onset of the fertility decline in 1910 according to the data of Reher (2004), yet their economic growth trajectories differed markedly throughout the 20th century. Similarly, both China and the People's Republic of Korea had the onset of the fertility decline in 1970, yet their economic development afterwards was vastly different. In our paper, we argue that institutional extractiveness provides the missing piece to solve this puzzle and we propose a full general equilibrium model that incorporates an endogenous institutional setting of a country and allows for the analysis of its impact on industrialization.

That the strength and inclusiveness of institutions could play a major role in economic development is well-known (see, for example, Sokoloff and Engerman, 2000; Acemoglu et al., 2001, 2002, 2005; Acemoglu and Robinson, 2012; Michalopoulos and Papaioannou, 2020; Papaioannou, 2025). Whenever property rights are underdeveloped and the ruling elite can extract resources from the population, incentives for entrepreneurship and innovation tend to suffer. Since technological progress is the driving force that propels the shift of households' preferences towards educating their children and since this very technological progress depends on proper incentives, it is clear that institutional quality must also have played a major role in driving the takeoff to sustained long-run growth.

While many other previously described forces have been analyzed within Unified Growth Theory, the role of institutions is difficult to embed in such a framework. The aim of our paper is to close this gap and provide a theory of the takeoff to industrialization that explicitly includes a ruling elite that optimally extracts resources from output/income, which reduces entrepreneurship, and from innovation, which reduces the forces of creative destruction and, thus, of long-run technological progress. This allows us to model much more differentiated effects of extractive institutions according to which the long-run development impacts depend on the underlying decisions of the elite from where to extract resources. A key insight of our framework is that the nature of extractive opportunities evolves endogenously with economic development. In the pre-industrial phase, elites can only extract from output. But as the economy industrializes and firms begin conducting R&D, a new extraction channel opens—one that, as we show, has permanent growth effects. This dynamic interaction between structural transformation and elite behavior cannot be captured by models that do not distinguish explicitly between development phases.

To allow for the distinction between the effects of extraction from output/income and extraction from innovation, we follow Peretto (2015) and Chu et al. (2022, 2024) and propose a model with two phases of industrialization. In the first phase, economic growth is driven by horizontal innovation, that is, the creation of new firms and product varieties. In the second phase, vertical innovation, that is, improvements in the quality of existing products, complements variety expansion as engine of growth. We then introduce a ruling elite that extracts resources from the population subject to the constraint of being able to stay in power. The more extractive institutions are and, thus, the lower the protection of property rights is, the more resources the ruling elite can extract. We show that extraction of parts of output/income mainly reduces incentives for entrepreneurship. This, in turn, reduces firm creation and horizontal innovation, delays the takeoff to sustained economic growth, and reduces economic growth during the first phase of industrialization. However, the long-run economic growth rate is not affected by this type of extractive institutions, which is fully consistent with the findings of Chu et al. (2022). As a second dimension of the extractiveness of institutions, we allow them to hamper creative destruction by depriving resources meant for vertical innovation. We show that this form of extractive institutions has a more damaging effect on long-run economic growth, while it does not affect the timing of the takeoff.

Overall, extracting innovation resources that could lead to creative destruction may be less costly and more effective for the ruling elite than extracting part of the output/income of the whole population. First, this is because output extraction affects a large part of the population and could therefore spark strong resistance. Second, extraction from innovation could reduce creative destruction, a process that is often by itself seen as a threat to the ruling elite (Acemoglu and Robinson, 2012). Examples on the two forms of extraction and their difference in terms of resistance of the population abound. For example, the grain requisitioning during collectivization in the Soviet Union can clearly be classified as extraction of output and had severe economic and social consequences. It caused famine (e.g., the Holodomor in Ukraine) and unrest, which, in turn, led to brutal repression (see also Conquest, 1986; Markevich et al., 2021). Other examples for such output extraction may be the serfdom system in Europe and the tribute systems of Spanish conquerors in Latin America, which are likely to have delayed economic development (Acemoglu and Robinson, 2012). By contrast, the suppression of some forms of scientific innovation in the Soviet Union, such as banning Mendelian genetics in favor of pseudo-scientific Lysenkoism—which was more in line with Marxism—destroyed genetic research in the Soviet Union and caused long-term damage to innovation and productivity. However, it did not provoke mass opposition (Graham, 1993) and, thus, was politically "cheaper" than output extraction. The slow adoption of the printing press in the Ottoman Empire until the 18th Century has also been attributed to the fear of the ruling elite to lose power and revenues (Coşgel et al., 2012). Another early and consequential example of the differences between the two types of extraction is related to the policies of the Qing dynasty in China. The suppression of commerce, high taxation, and forced grain procurement sparked rural uprisings (e.g., the White Lotus Rebellion). However, it was arguably the neglect of innovation and open science (e.g., neglecting printing technologies, the modern scientific method, or the international exchange of ideas) that did most long-run damage to the economy, while facing little immediate resistance (cf. Landes, 1998; Pomeranz, 2001).

In the quantitative analysis, we discipline the model by calibrating it to U.S. data over the 1650–2022 period and use the resulting framework to assess the magnitude of the growth effects implied by extractive institutions. The model endogenously identifies a takeoff to industrialization that is in line with the results of Iacopetta and Peretto (2021) and reproduces the observed post-industrialization growth patterns in the data. We then perform counterfactual experiments that illustrate our analytical findings. Output extraction primarily delays the industrial takeoff and depresses growth during the transition. However, it leaves the long-run growth rate unaffected. By contrast, extraction from innovation activities substantially reduces growth during industrialization and permanently lowers the balanced growth rate. Quantitatively, R&D is able to explain the bulk of growth differences across the counterfactual simulations that we consider. This once more underscores the persistent development costs of innovation-suppressing institutions.

Our contribution is related to the following previous literature. O'Rourke et al. (2013) and Lehmann-Hasemeyer et al. (2023) include basic scientific knowledge in the production function of new ideas in an endogenous growth framework of the Romer (1990)–Jones (1995) type. They show that factors that hamper basic scientific knowledge creation, such as anti-scientific ruling elites, delay the transition to modern economic growth substantially (Wootton, 2015; Mokyr, 2016). Lagerlöf (2016, 2021) proposes models for extractiveness of institutions and endogenous statehood formation in which multiple equilibria can occur. Specifically, Lagerlöf (2021) shows that democratic states tend to grow faster than authoritarian ones, which, in turn, tend to grow faster than non-states (before the onset of statehood formation). Our paper is most closely related and builds on Peretto (2015) and Chu et al. (2024) who show the importance of horizontal versus vertical innovation and of government spending on infrastructure in explaining industrialization and the transition to long-run economic growth; and Iacopetta and Peretto (2021) who study an endogenous takeoff in a Schumpeterian framework in which 'resource extraction' arises from corporate-governance frictions internal to innovative firms. The main difference to their approach is that we do not take the distortion as given. Instead, we model a ruling elite that chooses where to extract—either from output or from innovation activity—and we derive this choice from a political survival constraint. Finally, our paper is closely related to Chu et al. (2022), who analyze how output extraction affects the timing of the takeoff to sustained growth. We build on their approach and introduce a ruling elite that decides endogenously how much to extract and whether they target output/income or the resources that are used for innovation, which hampers creative destruction. We show that this distinction is very important in explaining differential effects of extractive institutions and that extractive institutions may not only affect the timing of the takeoff and economic growth in the short-run, but they may also strongly impact upon economic growth in the long-run. To summarize, our central contributions are: (i) to introduce the effect of institutions on industrialization and the takeoff to sustained economic growth in a full-fledged general equilibrium endogenous economic growth model, (ii) to endogenize the extractiveness of institutions and elite behavior under a political survival constraint, and (iii) to distinguish between extraction from output and extraction from innovation, which are conceptually different and also different in their impact on the takeoff on the one hand and on long-run economic growth on the other.

The article is structured as follows. In Section 2, we describe the basic model setup. Section 3 is devoted to the analysis of the effects of resource extraction by the ruling elite. Section 4 contains the analytical results on the effects of extraction by the elite on the timing of the takeoff to the industrial phase, the growth rate during industrialization, and the long-run balanced growth rate of the economy. In Section 5, we calibrate the model to U.S. data and simulate the impacts of different extraction rates on economic development. Finally, we conclude and provide policy recommendations in Section 6.

2. The model

We introduce endogenous extraction by a ruling elite in the Schumpeterian model of industrialization with an endogenous takeoff to sustained long-run economic growth as introduced by Peretto (2015) and extended by Chu et al. (2022).[1] The model incorporates two types of innovation: the development of new products (market entry of entrepreneurs) and the improvement of existing ones (creative destruction). By combining these two dimensions, the model generates an endogenous market structure, which eliminates the strong scale effect of standard R&D-based growth models. Workers earn wages to consume or save, while the ruling elite extracts resources from output and/or from R&D inputs. Different extraction choices of the ruling elite generate different effects on: (i) the timing of the takeoff to sustained growth, (ii) the industrialization phase, and (iii) on the long-run balanced growth path.

2.1. Households

The economy is closed and consists of a continuum of infinitely-lived households of mass one. Households are divided into two types: the ruling elite and workers, indexed by $i = \{e, w\}$, respectively. Both types of households derive their lifetime utility from consumption of a final good and each type is endowed with one unit of time. We assume, for simplicity, that all households have a logarithmic utility function:

$$u^i(c_t) = \ln(c_t^i). \tag{1}$$

Household i chooses its per capita consumption c_t^i to maximize the present value of lifetime utility

$$U_0^i = \int_0^\infty e^{-(\rho-\lambda)t} \ln(c_t^i)\, dt, \tag{2}$$

[1] Other models of endogenous or semi-endogenous economic growth such as the ones of Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) and Jones (1995), or Kortum (1997) lack the industrial structure of the Peretto (2015) model that allows us to differentiate in detail between output extraction and innovation extraction.

where $\rho > 0$ is the subjective time discount rate and $\lambda > 0$ refers to the rate of population growth. The population size at time t is then given by

$$L_t = L_0 e^{\lambda t},$$

where we normalize the initial population size to $L_0 = 1$, without loss of generality. To ensure finite lifetime income, we impose the standard parameter restriction $\rho - \lambda > 0$.

Workers own financial assets and supply their time endowment inelastically in the labor market. The per capita budget constraint of a representative worker is

$$\dot{a}_t^w = (r_t - \lambda)a_t^w + w_t - c_t^w, \tag{3}$$

where a_t^w denotes financial assets per worker, c_t^w is per capita consumption of workers, w_t is the wage rate, and r_t refers to the interest rate. Thus, the representative worker chooses consumption at each point in time to maximize lifetime utility (2), subject to the budget constraint (3). Dynamic optimization yields the optimal consumption path of workers as

$$\frac{\dot{c}_t^w}{c_t^w} = r_t - \rho, \tag{4}$$

which is the well-known consumption Euler equation.

The second type of households represents the politically dominant ruling elite. The ruling elite do not supply labor but, instead, they extract a constant fraction $\gamma_y \in (0,1)$ of aggregate output, Y_t and/or a constant fraction $\gamma_Q \in (0,1)$ of R&D inputs, Q_t. The elite earns what they extract, such that

$$w_t^e = \gamma_y Y_t + \gamma_Q Q_t,$$

and their asset accumulation is described by

$$\dot{a}_t^e = (r_t - \lambda)a_t^e + w_t^e - c_t^e. \tag{5}$$

Carrying out dynamic optimization leads to the consumption Euler equation

$$\frac{\dot{c}_t^e}{c_t^e} = r_t - \rho, \tag{6}$$

which implies that the ruling elite has the same optimal consumption growth path as the workers. This is reasonable because they share the same time preference rate and the same interest rate. However, the income level (through extraction) and, thus, the asset level of the elite are higher than the corresponding levels of workers. Thus, the elite can afford a higher consumption *level*.

Overall, the elite would strive to maximize their income level in terms of extraction. The corresponding optimization problem is to maximize income by choosing a combination of optimal extraction shares (γ_y, γ_Q) at every instant of time, subject to a political survival constraint. This constraint reflects the fact that the extraction process incurs costs (which we model quadratically) in terms of a threat to the elite's position in power. The tradeoff is clear: while higher extraction raises the elite's rents, it also increases the probability that they lose power (see also Mizuno et al., 2017). The elite therefore needs to balance the extraction of resources against the risk of losing power. Assuming that the ruling elite wants to limit the risk of losing power below the level Ψ, the "keeping in power constraint" is given by

$$\kappa_1 \gamma_y + \kappa_2 \gamma_y^2 + \beta_1 \gamma_Q + \beta_2 \gamma_Q^2 \leq \Psi,$$

where κ_1, β_1, κ_2, and β_2 represent linear and nonlinear cost scaling parameters. Extracting innovation resources that could lead to creative destruction may be less costly to the ruling elite than extracting a part of income of the whole population. This is because (i) extraction from R&D inputs is less salient to the representative household than extraction from aggregate output, as it operates indirectly through growth rather than current income, and (ii) creative destruction is itself often seen as a threat by ruling elites, so that extracting from innovation-related resources serves a dual function.

The optimization problem can then be formulated using a Lagrangian function

$$L = \gamma_y + \gamma_Q + \chi(\Psi - \kappa_1 \gamma_y - \kappa_2 \gamma_y^2 - \beta_1 \gamma_Q - \beta_2 \gamma_Q^2), \tag{7}$$

where $\kappa_1 > \beta_1$ and $\kappa_2 > \beta_2$. These inequalities capture the notion described above that any given increase in extraction of resources used as input in R&D is politically less costly than the same increase in extracting output/income. The first-order conditions with respect to γ_y and γ_Q are

$$\frac{\partial L}{\partial \gamma_y} = 1 - \chi(\kappa_1 + 2\kappa_2 \gamma_y) = 0, \tag{8}$$

$$\frac{\partial L}{\partial \gamma_Q} = 1 - \chi(\beta_1 + 2\beta_2 \gamma_Q) = 0, \tag{9}$$

and the binding "keeping in power constraint" is

$$\kappa_1 \gamma_y + \kappa_2 \gamma_y^2 + \beta_1 \gamma_Q + \beta_2 \gamma_Q^2 = \Psi. \tag{10}$$

Solving the system (8)–(10) yields closed-form solutions for the two pairs of candidate solutions. The solutions providing the economically meaningful (positive) extraction rates are

$$\gamma_y^* = \frac{\sqrt{\left(\kappa_2+\beta_2\right)\left(\kappa_1^2\beta_2+\kappa_2\left(\beta_1^2+4\beta_2\Psi\right)\right)}-\kappa_1\left(\kappa_2+\beta_2\right)}{2\kappa_2\left(\kappa_2+\beta_2\right)}, \tag{11}$$

and

$$\gamma_Q^* = \frac{\sqrt{\left(\kappa_2+\beta_2\right)\left(\kappa_1^2\beta_2+\kappa_2\left(\beta_1^2+4\beta_2\Psi\right)\right)}-\beta_1\left(\kappa_2+\beta_2\right)}{2\beta_2\left(\kappa_2+\beta_2\right)}. \tag{12}$$

Under $\kappa_1 > \beta_1$ and $\kappa_2 > \beta_2$, it follows from Eqs. (11)–(12) that $\gamma_Q^* > \gamma_y^*$. The optimal choice of the ruling elite is a preference for a higher R&D extraction share. Political survival constraints make high levels of extraction costly, but reducing creative destruction is politically more beneficial so that the ruling elite will optimally choose a higher share γ_Q^* than γ_y^*.[2]

The framework that we follow here is kept deliberately simple but nevertheless it captures the decisions that matter for the elite and, particularly, the effects that these decisions have on the underlying economy. However, many extensions of the framework are possible to increase its realism and to provide more microfoundations. For example, the interactions between workers and the elite could be modeled in terms of a political economy model in which the tolerance of workers for extraction followed a certain distribution. Whenever the extraction by the elite becomes so high that a majority of the population would not tolerate extraction anymore, the threat of a coup increases discontinuously. In the case of a democratically organized system but with a corrupt elite, the median voter would at some point cease to tolerate extraction and a political majority would force the ruling elite out of power. In reduced form, we get a similar behavior of the elite as in such systems when we interpret Ψ as the probability of the elite being removed from power.

2.2. Final output producers

A set of atomistic firms produce final goods in a perfectly competitive environment using the production technology

$$Y_t = \int_0^{N_t} X_t(i)^{\theta}\left[Z_t(i)^{\alpha} Z_t^{1-\alpha}\frac{L_t}{N_t^{1-\sigma}}\right]^{1-\theta} di, \tag{13}$$

where N_t is the number of differentiated varieties used as inputs in production, $X_t(i)$ is the quantity of non-durable intermediate product i used at every instant of time t, and $Z_t(i)$ is the quality level of that intermediate product. The parameters $(\theta,\alpha) \in (0,1)$ determine the elasticity of substitution between differentiated intermediate goods, and the private return to quality, respectively. Thus, the term $(1-\alpha)$ refers to the knowledge spillovers across industries. The parameter $\sigma \in (0,1)$ reflects a congestion effect in the use of existing differentiated intermediate goods for producing output: using more differentiated varieties as inputs increases output, but less than proportionately. The productivity of each intermediate good depends on the specific quality level of that product, $Z_t(i)$, and on the technological spillovers from the average quality, $Z_t = \frac{1}{N_t}\int_0^{N_t} Z_t(i)\,di$.

Final goods producers maximize profits

$$\Pi_t = (1-\gamma_y)Y_t - w_t L_t - \int_0^{N_t} P_t(i)X_t(i)\,di \tag{14}$$

at each instant, where $\gamma_y \in (0,1)$ is the extraction rate on final output imposed by the ruling elite and $P_t(i)$ is the price of intermediate good i. In a decentralized equilibrium, instantaneous profits are zero because of perfect competition. From (14), we can derive the conditional demand functions for productive labor and intermediates as

$$w_t = (1-\gamma_y)(1-\theta)\frac{Y_t}{L_t}, \tag{15}$$

$$X_t(i) = \left[\frac{(1-\gamma_y)\theta}{P_t(i)}\right]^{\frac{1}{1-\theta}} \frac{Z_t(i)^{\alpha} Z_t^{1-\alpha} L_t}{N_t^{1-\sigma}}, \tag{16}$$

where firm size $X_t(i)$ decreases in the extraction rate γ_y, but is linear in the ratio of the population size to the number of intermediate products, $\frac{L_t}{N_t^{1-\sigma}}$. Given the competition structure, all output is paid to the factor inputs as income such that

$$w_t L_t = (1-\gamma_y)(1-\theta)Y_t, \tag{17}$$

$$\int_0^{N_t} P_t(i)X_t(i)\,di = (1-\gamma_y)\theta Y_t. \tag{18}$$

[2] Setting $\kappa_1 = \beta_1$ and $\kappa_2 = \beta_2$ implies that the ruling elite has identical preferences with respect to extraction from output or from R&D inputs. From Eqs. (11)–(12), it then follows that $\gamma_y = \gamma_Q$. Setting $\kappa_1 = \kappa_2 = 0$ and $\beta_1 = \beta_2 = 0$ yields the corner solution cases in which the ruling elite introduces a single source of extraction (either on output or on R&D inputs).

2.3. Intermediate firms

Intermediate firms are monopolistically competitive and have two roles in this economy: they produce existing intermediate goods so as to maximize the value of the monopolistic firm, and they do in-house R&D to improve the intermediates. Thus, intermediate firms not only produce non-durable goods for use as inputs in final goods production, but also improve the quality of those intermediate inputs.

The profit flow of the firm producing intermediate i before investing in R&D is

$$\pi_t(i) = [P_t(i) - 1]X_t(i) - \phi Z_t(i)^{\alpha} Z_t^{1-\alpha}, \tag{19}$$

where, following Peretto (2015), we assume that firms producing intermediate goods operate with a one-for-one technology. They use final output as input to produce intermediate goods with a unit marginal cost. In addition, intermediate firms face a fixed operating cost $\phi Z_t(i)^{\alpha} Z_t^{1-\alpha}$ measured in terms of labor, which can be interpreted as administrative/managerial cost (Peretto and Connolly, 2007).[3]

Intermediate firms engage in in-house R&D to improve the quality of existing intermediate goods. In so doing, the firm devotes $Q_t(i)$ units of final output to R&D, such that

$$\dot{Z}_t(i) = (1 - \gamma_Q)Q_t(i), \tag{20}$$

where $\gamma_Q \in (0,1)$ is the extraction rate that the ruling elite imposes on the units of final output used as input in in-house R&D. The value of firm i is then given by

$$V_t(i) = \int_t^{\infty} e^{-\int_t^u r_s\, ds}[\pi_u(i) - Q_u(i)]\, du, \qquad u \geq t, \tag{21}$$

where $V_t(i)$ is the present value of all future profits less the cost incurred for in-house R&D from the instant of time t—in which the patent for that product was obtained—onwards.

We follow Chu et al. (2020a,b) and instead of the incumbent engaging in Bertrand pricing, we assume that the other competitors in each industry i can produce an intermediate good of the latest quality $Z_t(i)$, but they have to incur a higher unit cost of production, given by $\mu > 1$. Therefore, to get priced out of the market, the free entry condition in the vertical innovation market implies that $P_t(i) = \mu$. Then, monopolistic firms maximize (21) given (20) and (16), taking into account the constraint that this price is binding.[4]

This class of models is characterized by symmetry in equilibrium, implying that intermediate firms start with the same initial quality and quantity, charge the same price, which determines the value and profits of the firm, and make identical decisions. According to Peretto (1998, 1999), the conditions for symmetry are that the firm-specific return to quality innovation decreases in the specific quality level $Z_t(i)$ and that entrants enter at the average quality level Z_t. Both conditions are fulfilled in this strand of models. Furthermore, the crucial variable for analyzing the dynamics of the economy over time is the size of firms, which determines the incentives to innovate. Using (16), assuming symmetry, and given that $P_t = \mu$, the quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \left[\frac{(1-\gamma_y)\theta}{\mu}\right]^{\frac{1}{1-\theta}} \frac{L_t}{N_t^{1-\sigma}}, \tag{22}$$

where, as expected in the baseline Schumpeterian model, firm size is linearly increasing in population size and decreasing in the number of firms. In addition, firm size is also decreasing in the output extraction rate. To simplify the dynamic analysis further, we define a variable that maintains the same dynamics as the quality-adjusted firm size. There are several ways in which this variable can be defined and we set

$$x_t = \left[\frac{\mu}{1-\gamma_y}\right]^{\frac{1}{1-\theta}} \frac{X_t}{Z_t} = \theta^{\frac{1}{1-\theta}} \frac{L_t}{N_t^{1-\sigma}}. \tag{23}$$

Log-differentiating the newly transformed variable in (23) with respect to time yields

$$\frac{\dot{x}_t}{x_t} = x_t - z_t = \lambda - (1-\sigma)n_t, \tag{24}$$

where we follow the traditional notation and use lowercase letters to denote the growth rate of variables ($\frac{\dot{X}_t}{X_t} = x_t$ and $\frac{\dot{Z}_t}{Z_t} = z_t$). In Appendix C, we show that the dynamics of the state variable is globally stable provided that the parameter restriction

$$\delta(1-\gamma_Q)\phi > \frac{1}{\alpha}\left[\mu - 1 - \delta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right] > \mu - 1 \tag{25}$$

holds at every instant, where $\delta > 0$ refers to the cost of developing a new variety in terms of final output (the entry cost). Given this restriction, starting from an initial value x_0, the state variable x_t will gradually increase towards its steady-state value x^*.

[3] Following Peretto (2015), this result is robust to including different spillover effects for the private return to quality, α, and vertical spillovers, $1-\alpha$.

[4] The technical details are presented in Appendix A.

Using expression (A.5) in Appendix A, the rate of return on vertical innovation can then be obtained as

$$r_t^q = (1-\gamma_Q)\alpha\left[(\mu-1)\left(\frac{1-\gamma_y}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right]. \tag{26}$$

We observe that it is reduced by both rates of extraction, γ_Q and γ_y.

2.4. R&D firms

The economy features both horizontal innovation, driven by the expansion of differentiated varieties through new market entrants, and vertical innovation, driven by in-house R&D through intermediate monopolistic firms. As described above, we assume that developing a new variety requires a sunk cost of δX_t units of final output.

The no-arbitrage condition, which holds regardless of whether new firms are entering the horizontal innovation market, is given by

$$r_t = \frac{\pi_t - Q_t}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{27}$$

It states that the rate of return on investment needs to equal the dividend yield (the first term on the right-hand side) and the valuation gain of the firm (the second term on the right-hand side).

When entry occurs, the free-entry condition requires that

$$V_t = \delta X_t, \tag{28}$$

where the setup cost is increasing in the initial output volume of the firm. This implies that entrants are willing to pay the discounted future profit stream as fixed up-front investment. If fixed costs were lower, firms would have an incentive to enter the market, which would drive profits down to such an extent that the free-entry condition would again be fulfilled with equality.

Using (19), (22), (27), and (28), the rate of return to horizontal innovation is

$$r_t^e = \frac{1}{\delta}\left[(\mu-1) - \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{\phi(1-\gamma_Q)+z_t}{(1-\gamma_Q)x_t}\right] + z_t + \frac{\dot{x}_t}{x_t}. \tag{29}$$

Again, we observe that both rates of extraction feature in this expression.

2.5. Aggregation and equilibrium

To derive the aggregate level of output, we substitute $P_t = \mu$ and (16) into (13) to get

$$Y_t = \left[\frac{(1-\gamma_y)\theta}{\mu}\right]^{\frac{\theta}{1-\theta}} N_t^{\sigma} Z_t L_t, \tag{30}$$

where the level of output is decreasing in the extraction rate of final output. If we define per capita output as $y_t \equiv \frac{Y_t}{L_t}$, then log-differentiating (30) with respect to time yields the per capita growth rate of output

$$g_t = \sigma n_t + z_t, \tag{31}$$

where $g_t = \frac{\dot{y}_t}{y_t}$ and $n_t = \frac{\dot{N}_t}{N_t}$ (see Peretto, 1998, 2015). The growth rate of output per capita is determined by the growth rate of quality z_t and the growth rate of variety n_t.

We can now define the equilibrium of the economy.

Definition 1. The equilibrium in this economy is defined as a time path of allocations (a_t, c_t^w, c_t^e, Y_t, Q_t, and X_t) and prices (r_t, P_t, w_t, V_t) such that:

- households maximize lifetime utility taking r_t and w_t as given;
- the representative "ruling elite" household's income is given by its extraction $\gamma_y Y_t + \gamma_Q Q_t$;
- final goods producers are perfectly competitive and maximize profits, taking P_t and w_t as given;
- intermediate monopolistic firms choose $\{P_t, Q_t\}$ given the quantity demanded of intermediate i, and maximize V_t, taking r_t as given;
- entrant firms make decisions to enter the R&D market, given V_t;
- being a closed economy, the total value of assets equals the total value of firms, $a_t L_t = N_t V_t$;
- the final output market clears in the pre-industrial era: $Y_t = c_t^w L_t + c_t^e L_t + \mu N_t X_t$;
- the final output market clears in the industrial era: $Y_t = c_t^w L_t + c_t^e L_t + N_t(X_t + \phi Z_t + Q_t) + \dot{N}_t \delta X_t$.

3. Extraction of resources, endogenous takeoff, and long-run growth

The dynamic evolution of the economy is driven by the (quality-adjusted) firm size x_t that provides incentives for innovation (see Laincz and Peretto, 2006, for empirical evidence). The economy is initially in a pre-industrial era, where innovation (both horizontal and vertical) is zero, because firm size is too small to provide sufficient incentives. Once quality-adjusted firm size becomes sufficiently large, the economy enters the first phase of the industrial era, where horizontal innovation appears first and the range of varieties grows.[5] After creating a new variety and for a sufficiently large firm size there is an incentive to improve the quality of the variety, such that vertical innovation starts in addition to horizontal innovation and the economy enters the second phase of the industrial era. Peretto (2015) shows that the occurrence of either growth engine produces the same convex–concave profile of the long-run per capita output growth rate, from an acceleration phase to a deceleration and eventually convergence from below to a stationary growth rate. This is what Peretto aptly calls the "stairway to growth" (Peretto, 2026, pp. 135–140), a terminology that we also follow below when describing the corresponding figures.

3.1. Pre-industrial era

In the pre-industrial era, the firm size x_t is insufficient to activate innovation. Therefore, the growth rate of output per capita is

$$g_t = \sigma n_t + z_t = 0 \tag{32}$$

because $n_t = z_t = 0$. This situation captures well the stagnation of economies before the takeoff to sustained economic growth, although our mechanism for the takeoff differs from the quantity-quality tradeoff that usually occupies center stage in modeling the escape from Malthusian stagnation (Galor and Weil, 2000). Thus, we view our mechanism as a complementary force that operates in addition to the standard forces outlined in Unified Growth Theory (Galor, 2005, 2011).

The pre-industrial era ends when the present value of monopolistic firms becomes sufficiently large such that the free-entry condition in (28) is satisfied, or equivalently (21) holds with equality or exceeds it. Using (24) and (32), and taking into account that the starting value of N_t is N_0 in this period, the dynamics of the state variable x_t is given by

$$\frac{\dot{x}_t}{x_t} = \lambda, \tag{33}$$

which implies that the state variable grows at the rate of population growth. As population size increases over time, firm size x_t increases proportionally and eventually becomes sufficiently large to activate innovation.

During the pre-industrial era, the consumption-to-output ratio remains constant because no innovation occurs:

$$\frac{c_t^w}{y_t} = (1-\gamma_y)(1-\theta). \tag{34}$$

3.2. The first phase of the industrial era

In the first phase of the industrial era, firm size becomes sufficiently large so that horizontal innovation activates, while vertical innovation does not ($n_t > 0, z_t = 0$). Extraction of output by the ruling elite delays the start of the industrialization phase. Intuitively, increasing the extraction rate of output reduces resources otherwise intended for firms, which hampers their growth.

In the first phase of the industrial era, the rate of return on horizontal innovation is the prevailing interest rate in the economy. Since $z_t = 0$, it follows that $r_t^e = r_t$. From (B.5) in Appendix B, c_t^w/y_t is constant, hence $\dot{c}_t^w/c_t^w = \dot{y}_t/y_t = g_t$. Using (4), in the first phase $r_t = \rho + g_t = \rho + \sigma n_t$. Substituting this expression and (24) in (29), while holding $z_t = 0$, yields

$$n_t = \frac{1}{\delta}\left[\mu - 1 - \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{\phi}{x_t}\right] + \lambda - \rho. \tag{35}$$

The growth rate of the variety of goods depends negatively on output extraction by the ruling elite. As such, (35) can take both negative and positive values. Our interest lies at the critical point at which horizontal innovation starts to occur from stagnation. The growth rate of horizontal innovation is positive if and only if it rises above a threshold that we can derive from (C.2) in Appendix C as

$$x_N = \left[\frac{\mu}{1-\gamma_y}\right]^{\frac{1}{1-\theta}} \left[\frac{\phi}{\mu - 1 - \delta(\rho-\lambda)}\right] > x_0. \tag{36}$$

Once x_N rises above this threshold, horizontal innovation starts. Higher rates of extraction increase x_N, thus causing a delay in the industrialization process. If the ruling elite extracts very heavily, that is, when $\gamma_y \to 1$, the threshold in (36) converges to infinity, $x_N \to \infty$, and industrialization is prevented altogether. We summarize this finding in the following proposition.

[5] We follow Chu et al. (2020b) and present the more realistic case in which horizontal innovation precedes vertical innovation. In principle it is possible that vertical innovation appears first, but this is not the intuitively appealing case and it is also hard to square with the data.

Proposition 1. *If the ruling elite extracts very heavily from output, it will prevent the process of industrialization.*

Proof. Follows immediately from investigating Eq. (36) and the discussion given in the text. □

If the ruling elite does not extract so much that it prevents industrialization altogether, the growth rate of output per capita follows directly from $g_t = \sigma n_t$ and (35) as

$$g_t = \frac{\sigma}{\delta}\left[\mu - 1 - \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{\phi}{x_t}\right] + \sigma(\lambda - \rho) > 0. \tag{37}$$

The growth rate of output per capita decreases in the presence of increased extraction of output at every instant in this phase. Therefore, in addition to delaying the industrial takeoff, extraction of final output dampens the growth rate of per capita output throughout the first phase of the industrial era. The dynamics of x_t is given by

$$\dot{x}_t = \frac{1}{\delta}\left[(1-\sigma)\left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \phi - [(1-\sigma)(\mu - 1 - \delta\rho) - \delta\rho\lambda]x_t\right] > 0, \tag{38}$$

which is positive, indicating that firm size gradually increases throughout this phase, eventually allowing for vertical innovation and the transition to the second phase of the industrial era.

3.3. The second phase of the industrial era

In the second phase of the industrial era, firm size is sufficiently large to incentivize the quality improvement of existing varieties. Thus, both types of innovation co-exist in this phase of the industrial era, providing two different engines of economic growth ($n_t > 0$ and $z_t > 0$). The vertical innovation engine kicks in once $x_Z > x_N$, where this threshold is given by

$$x_Z \equiv \underset{x}{\text{arg solve}}\left\{ \frac{\alpha(1-\gamma_Q)(\mu-1)}{\left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}}} x + \frac{\sigma\phi\left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}}}{\delta x} = \alpha\phi(1-\gamma_Q) + \rho + \frac{\sigma(\mu-1)}{\delta} - \sigma(\rho - \lambda) \right\} > x_N$$

as shown in (C.8) in Appendix C. We then have $z_t > 0$ and the rate of return on vertical innovation is $r_t^q \equiv r_t = \rho + g_t$. Substituting (26) in this expression yields

$$g_t = (1-\gamma_Q)\alpha\left[(\mu - 1)\left(\frac{1-\gamma_y}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right] - \rho > 0. \tag{39}$$

The growth rate of output per capita decreases in both the extraction rate of output γ_y and the extraction rate of R&D inputs, γ_Q. This stands in contrast to the first phase of the industrial era, in which only γ_y slowed growth.

The economic growth rate is positive and continues to increase at every instant because the quality-adjusted firm size x_t increases. The growth rate of the variety of differentiated products is

$$n_t = \frac{1}{\delta}\left[\mu - 1 - \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{\phi(1-\gamma_Q) + z_t}{(1-\gamma_Q)x_t}\right] - \rho + \lambda > 0, \tag{40}$$

while the growth rate of the quality of varieties is given by

$$z_t = (1-\gamma_Q)\alpha\left[(\mu - 1)\left(\frac{1-\gamma_y}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right] - \rho - \sigma n_t > 0. \tag{41}$$

The linearized dynamics of the state variable x_t is given by (C.7) in Appendix C.

3.4. Balanced growth path

In the long run, firm size is constant, and converges to the steady-state value

$$x^* = \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{(1-\alpha)\phi - \left[\frac{\rho(1-\sigma)+\lambda\sigma}{(1-\gamma_Q)(1-\sigma)}\right]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \frac{\sigma\lambda}{1-\sigma}\right]} > x_Z. \tag{42}$$

From (24), the steady-state variety growth rate is

$$n^* = \frac{\lambda}{1-\sigma}. \tag{43}$$

Substituting (42) into (39) yields the long-run balanced growth rate of output per capita as

$$g^* = (1-\gamma_Q)\alpha\left[(\mu-1)\frac{(1-\alpha)\phi - \left[\frac{\rho(1-\sigma)+\lambda\sigma}{(1-\gamma_Q)(1-\sigma)}\right]}{(1-\alpha)(\mu-1)-\delta\left[\rho+\frac{\sigma\lambda}{1-\sigma}\right]} - \phi\right] - \rho > 0. \tag{44}$$

We observe that the long-run balanced growth rate of output per capita is positive and independent of γ_y, which is in line with Chu et al. (2022). On the other hand, and in contrast to Chu et al. (2022), the extraction from creative destruction, γ_Q, reduces the growth rate of the economy even in the long run.

4. Effects of extraction by the elite on economic growth

In this section, we discuss how different extraction rates affect the takeoff to industrialization, the growth dynamics in different phases, and the long-run economic growth rate per capita. We start from the general case and then we depart from it to characterize different special cases.

4.1. Optimal extraction shares ($\gamma_y > 0,\ \gamma_Q > 0,\ \gamma_Q > \gamma_y$)

In the presence of both extraction choices, the ruling elite prefers to extract more from R&D inputs because it is politically the less costly option. In the pre-industrial era, firm size grows constantly at the rate of population growth. Since firm size is still smaller than necessary to provide incentives for horizontal innovation, the economy fails to grow in terms of per capita output in this phase.

Once firm size exceeds a certain level ($x_N > x_0$), incentives appear for firms to start innovating, which marks the (industrial) takeoff of the economy. In the first phase of the industrial era, only horizontal innovation (creation of new designs) drives economic growth. Regardless of the presence of both extraction choices, in this phase the ruling elite can only extract from final output. Vertical innovation does not yet occur and no R&D inputs exist to extract from. Firm size decreases with an increasing extraction rate of output, γ_y, as more resources in the economy are used elsewhere. This causes a delay in the takeoff to the industrial era. In addition, the growth rate in the first phase of the industrial era decreases with a higher extraction rate on output throughout its dynamic path.

If the extraction rate on output imposed in the first phase is not prohibitively high, firm size will continue to grow at a positive rate until it reaches a point at which incentives for the improvement of the quality of existing varieties (vertical innovation) kick in. The economy enters the second phase of the industrial era in which both horizontal and vertical innovation determine the growth rate of the economy. Now, the ruling elite can choose either extraction rate. The optimal choice involves a higher extraction rate on R&D inputs. In this phase, the growth effects from extraction of output and R&D inputs are qualitatively identical *during the transitional dynamics*. However, because the ruling elite prefers a higher extraction rate on R&D inputs, γ_Q seems to have a stronger effect on the growth rate during the transitional path as well. Overall, both extraction rates affect the growth rate differently on the *balanced growth path*. While the long-run balanced growth rate per capita is invariant to changes in the extraction rate on output, γ_y, it decreases with a higher extraction rate on R&D inputs, γ_Q.

Fig. 1 provides a graphical representation of the dynamic paths of three economies as they pass through the different growth phases. They differ in the size of the extraction rates imposed, with the ordering ($\gamma_y < \gamma_y' < \gamma_y''$) and ($\gamma_Q < \gamma_Q' < \gamma_Q''$), going from left to right. The blue curve in the middle presents the optimal scenario from the perspective of the ruling elite, where $\gamma_Q > \gamma_y$. The curve on the far left has the lowest γ_y and the lowest γ_Q, while the curve on the far right has the highest γ_y and γ_Q. We observe that all curves follow a similar path: they start with a delay in the takeoff to the industrial era, and then they grow through both phases to settle at a different long-run growth rate per capita. However, the different combinations of imposed extraction rates can have different effects. The economy depicted by the far left curve experiences the shortest delay in the takeoff and the highest growth rates during the transitional dynamics in both phases of the industrial era. It also experiences the highest long-run growth rate per capita. On the other hand, the economy depicted by the far right curve experiences the longest delay in the takeoff and the lowest growth rates during the transitional dynamics in both phases of the industrial era. In addition, it experiences the lowest long-run growth rate. The blue (middle) curve depicts the scenario where $\gamma_Q > \gamma_y$, which holds for the optimally chosen scenario by the ruling elite. The results are summarized in the following proposition.

Proposition 2. *Let the ruling elite optimally choose a higher R&D extraction share relative to the output extraction share ($\gamma_Q > \gamma_y$). Then, the existence of $\gamma_y > 0$ delays the takeoff to the industrial era and reduces the growth rate during the transitional dynamics in the first phase of the industrial era. The optimal choice ($\gamma_Q > \gamma_y$) yields a stronger negative effect on the growth rate during the transitional dynamics in the second phase of the industrial era. Finally, the economy converges to a lower long-run growth rate per capita.*

Proof. See Appendix D. □

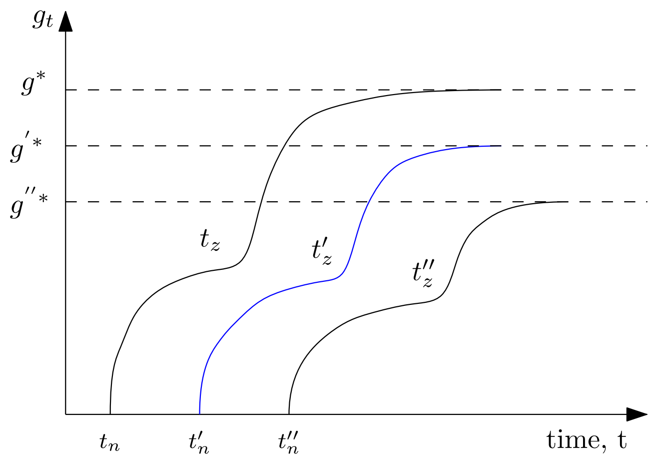

Fig. 1. Stairway to growth ($\gamma_y > 0$, $\gamma_Q > 0$ and $\gamma_Q > \gamma_y$).
Description. This figure shows the dynamic paths of three economies. The economy depicted by the curve on the far left has smaller extraction rates on output ($\gamma_y < \gamma'_y$) and R&D inputs ($\gamma_Q < \gamma'_Q$) than the economy depicted with the curve on the far right. The figure shows that this delays the start of the first phase of the industrial era ($t_n < t'_n$) and decreases the growth rate during this phase ($g_n > g'_n$). The start of the second phase of the industrial era is also delayed and a higher extraction rate of R&D inputs decreases the growth rate during the second phase of the industrial era ($g_z > g'_z$). In addition, the economy with a lower extraction rate converges to a higher long-run balanced growth rate per capita. The curve in the middle (the blue line) depicts the economy under endogenous extraction rates ($\gamma_Q > \gamma_y$). A smaller γ_y reduces the delay of the industrial era and increases the growth rate during the first phase of the industrial era. However, a higher γ_Q leads to a steeper downturn and a longer drag during the second phase of the industrial era as compared with either of the two other curves. Eventually, this economy converges to a lower long-run economic growth rate. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4.2. Identical extraction shares ($\gamma_y > 0$, $\gamma_Q > 0$, $\gamma_y = \gamma_Q$)

We briefly discuss the special case in which the ruling elite imposes both extraction shares but it has no particular preference towards either one. Here, the extraction shares are identical in size and the economy follows a similar dynamic behavior as in the general case.

In the pre-industrial era, there is no per capita output growth, although firm size keeps growing at the rate of population growth. Due to the presence of an extraction rate on output imposed by the ruling elite, the takeoff to the industrial era experiences a delay. If the extraction rate is higher (lower) than the preferred rate γ_y^*, the delay will be longer (shorter). The growth rate of the economy in the first phase of the industrial era decreases with higher extraction rates on output. Regardless, firm size continues to grow and eventually incentives appear for vertical innovation. In the second phase of the industrial era, the growth rate during the transitional dynamics and along the balanced growth path decreases with a higher extraction rate on R&D. The preference for a higher γ_Q in the optimal choice of the ruling elite will likely introduce a higher R&D extraction rate than in the identical extraction rates case. This impacts, in particular, the dynamics of the growth rate in the second phase of the industrial era, as with identical extraction shares, the negative effects on economic growth during the transition are milder. In addition, the long-run economic growth rate is also expected to be slightly higher. The results are summarized in the following proposition.

Proposition 3. *Let the ruling elite impose the same extraction rate on both final output and R&D inputs, that is, $\gamma_y = \gamma_Q = \gamma$. Then, a higher γ delays the takeoff to the industrial era, lowers the growth rate of per capita output during the first and second phases of the industrial era, and lowers the long-run balanced growth rate of the economy.*

Proof. See Appendix D. □

4.3. Corner cases ($\gamma_y > 0$, $\gamma_Q = 0$) and ($\gamma_y = 0$, $\gamma_Q > 0$)

We turn our attention to addressing the corner cases in which only one extraction rate is imposed by the ruling elite. We start with the special case in which the ruling elite introduces an extraction rate on output $\gamma_y > 0$ but not on R&D inputs $\gamma_Q = 0$. In the pre-industrial era, the growth rate remains invariant to any changes in γ_y and firm size continues to grow at the rate of population growth.

The imposed extraction rate on output raises the required firm size that provides sufficient incentives for horizontal innovation. Thus, a higher extraction rate of output delays the start of the industrial era. Once the first phase of the industrial era starts, the growth rate on the transitional path also decreases with a higher extraction rate on output. Since firm size grows at a constant rate,

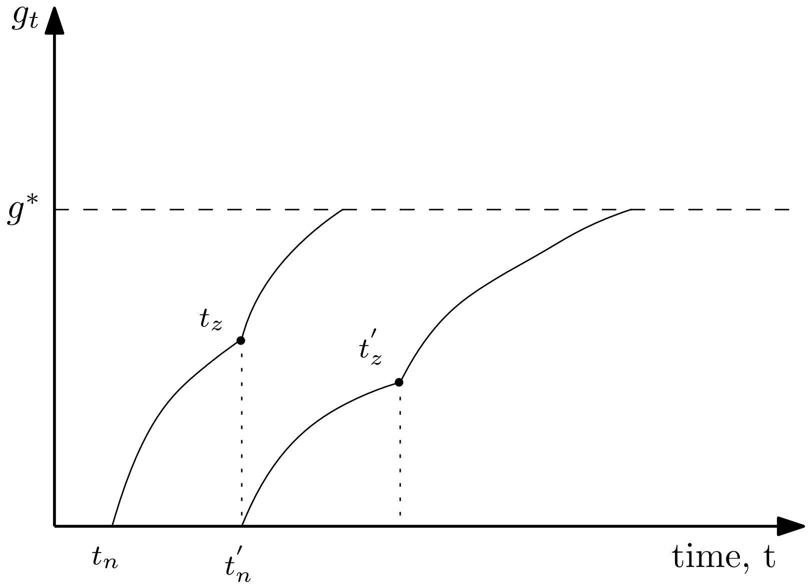

Fig. 2. Stairway to growth ($\gamma_y > 0$ and $\gamma_Q = 0$).
Description. This figure shows the dynamic path of two economies. The economy depicted by the curve on the left has a smaller extraction rate on output than the economy depicted by the curve on the right ($\gamma_y < \gamma_y'$). The figure shows that a higher extraction rate on output delays the start of the first phase of the industrial era ($t_n < t_n'$), decreases the growth rate during the first phase ($g_n > g_n'$), and delays the start of the second phase ($t_z < t_z'$). However, both economies converge to the same long-run balanced growth rate per capita.

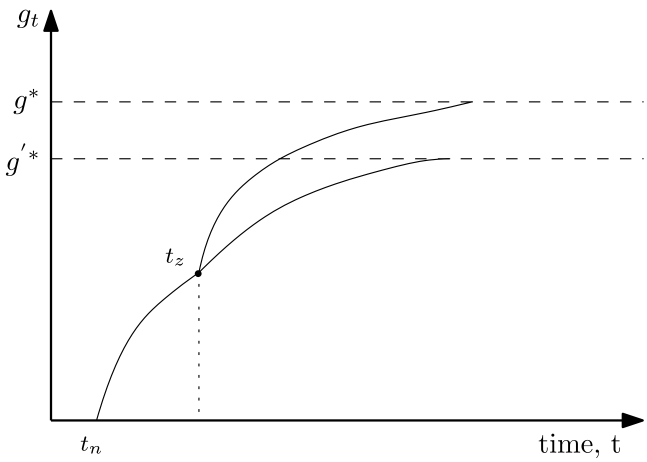

Fig. 3. Stairway to growth ($\gamma_y = 0$ and $\gamma_Q > 0$).
Description. This figure shows the dynamic path of two economies. Initially, both economies share the same transitional dynamics in the pre-industrial phase and the first phase of the industrial era ($t_n = t_n'$). Then, the different extraction rates on R&D inputs (as a share of output) γ_Q produce two different paths. The economy depicted by the curve on the left has a smaller extraction rate on R&D inputs than the economy depicted by the curve on the right ($\gamma_Q < \gamma_Q'$). The figure shows that a higher extraction rate on R&D inputs decreases the growth rate during the second phase of the industrial era ($g_z > g_z'$). In addition, the economy with a lower extraction rate converges to a higher long-run balanced growth rate per capita.

the more output is extracted by the ruling elite, the larger the firm size required to start the industrial phase. In addition to creating a delay in the start of the industrial era, a higher γ_y produces a negative growth effect during the transition of the economy in the first phase of the industrial era. A higher γ_y also decreases the growth rate during the transitional dynamics in the second phase of the industrial era. However, it has no effect on the long-run balanced growth rate.

Fig. 2 provides a graphical representation of the full dynamics of two economies that have only imposed extraction rates on output, $\gamma_y > 0$ and $\gamma_Q = 0$. The curve on the left is characterized by a smaller extraction rate on output. As a result, the takeoff to the industrial era occurs sooner than in the other economy (the curve on the right). Furthermore, the economy with a smaller

γ_y grows faster during the transition in both phases of the industrial era, although both economies converge to the same long-run balanced growth rate.

Next, we consider the other corner case in which the ruling elite imposes only an R&D extraction rate, $\gamma_y = 0$, but does not extract from output, $\gamma_Q > 0$. There are no notable changes in the pre-industrial era as compared to the previous scenarios.

In the absence of any extraction rate in the first phase of the industrial era, the economy can use all resources to the full extent. Thus, there is no delay in the takeoff of the industrial era, nor any growth slowdown during this phase. However, once firm size is sufficiently large to provide incentives for the start of vertical innovation, the ruling elite imposes an R&D extraction rate. During the second phase of the industrial era, the economic growth rate decreases with a higher R&D extraction rate both during the transitional dynamics and on the balanced growth path. Hence, the imposition of an R&D extraction rate produces negative long-run growth effects.

Fig. 3 provides a graphical representation of the full dynamics of two economies that have not imposed extraction rates on output, $\gamma_y = 0$, but only on R&D, $\gamma_Q > 0$. Initially, the takeoff and the first phase of the industrial era are identical for both economies. Then, once incentives appear for vertical innovation, the ruling elite starts to extract from R&D. In the second phase of the industrial era, the economy depicted by the curve on the left faces a smaller R&D extraction share. As such, it experiences faster economic growth both during the transitional dynamics and in the long run. Our results are summarized in the following proposition.

Proposition 4. *Consider the two "corner" regimes in which the ruling elite imposes only one extraction rate:*

1. *If $\gamma_y > 0$ and $\gamma_Q = 0$, then a higher γ_y delays the takeoff to the industrial era, reduces the per capita growth rate in both the first and second phases of the industrial era, but leaves the long-run balanced growth rate unchanged.*
2. *If $\gamma_y = 0$ and $\gamma_Q > 0$, then a higher γ_Q neither has an effect on the timing of the takeoff nor on economic growth in the first phase of the industrial era, but it reduces the growth rate in the second phase and on the long-run balanced growth path.*

Proof. See Appendix D. □

5. Quantitative analysis

We calibrate the model to the U.S. economy over the 1650–2022 period and use it to quantify how extraction rates γ_y and γ_Q affect the timing of the takeoff, the pace of industrialization, and the long-run economic growth rate. The calibration anchors the model to historical GDP per capita data from the Maddison Project Database (MPD) 2023, and targets long-run moments of the U.S. growth experience to discipline the technology and preference parameters of the model. Following Iacopetta and Peretto (2021), a late-sample fit (1990–2020) is solely used to select among parameter values that satisfy these target moments and feasibility conditions. We then conduct counterfactual experiments that vary the extraction rates while holding technology parameters fixed, isolating the direct extraction channels.

5.1. Calibration

The model features the parameter vector $\{\rho, \lambda, \gamma_y, \gamma_Q, \alpha, \theta, \sigma, \mu, \delta, \phi\}$ separated in two groups: parameters set externally from the literature and data sources and parameters chosen to jointly match long-run targets and the historical growth path as closely as possible. Table 1 reports the full set of values.

We set the discount rate at a conventional value of $\rho = 0.03$. The population growth rate $\lambda = 0.009$ follows closely the average U.S. growth rate since 1800. Letting $1 - \theta = 40\%$ pins down the intermediate goods share θ at 0.60. The elasticity of profits with respect to own knowledge is set at $\alpha = 0.333$ following Iacopetta and Peretto (2021). The markup ratio $\mu = 1.55$ falls within the range of estimates for the U.S. economy reported by De Loecker et al. (2020). In addition, the baseline parameter values must satisfy the inequality in Eq. (25) and the ordering $(x_0 < x_N < x_Z < x^*)$ for the dynamics of the model to hold. Let model time t be measured in years with $t = 0$ corresponding to calendar year 1800.[6] Hence calendar year T corresponds to $t = T - 1800$. When reporting dates (e.g., the takeoff year), we convert back to calendar time via $T = 1800 + t$.

The key parameters γ_y and γ_Q determine the extraction rates of the ruling elite on output and R&D inputs, respectively. As such, they play a vital role in the analysis of the timing of the takeoff and the transitional growth rate of the economy. For the baseline calibration, we consider a plausible range of values for $\gamma_y, \gamma_Q \in [0.01, 0.05]$. The elite's optimal extraction rates solely depend on the preference parameters $(\kappa_1, \kappa_2, \beta_1, \beta_2)$ and the political-risk threshold Ψ. To remain consistent with the qualitative analysis, we impose $\kappa_1 > \beta_1$ and $\kappa_2 > \beta_2$, which implies $\gamma_Q > \gamma_y$.

In the absence of empirically established values for the ruling elite's preference parameters, we proceed by reverse-calibrating the linear terms (κ_1, β_1) from target extraction rates. Specifically, we set $\gamma_y = 0.02$ and $\gamma_Q = 0.04$, choose the baseline nonlinear preference parameters as $(\kappa_2, \beta_2) = (1, 0.8)$, and set the baseline political-risk threshold to $\Psi = 0.10$. These extraction rates are below the lower end of the range $[0.07, 0.21]$ calibrated by Chu et al. (2022), reflecting the relatively inclusive institutional environment

[6] Although the dataset starts from 1650, there are only three data points between 1650 and 1800. Given the limited quantitative contribution of this period, we focus on the period 1800–2022.

of the U.S. economy. The counterfactual analysis below varies these parameters systematically. Using the first-order conditions and the binding keeping-in-power constraint, we obtain

$$m \equiv \frac{1}{\chi} = \frac{\Psi + \kappa_2 \gamma_y^2 + \beta_2 \gamma_Q^2}{\gamma_y + \gamma_Q}, \qquad \kappa_1 = m - 2\kappa_2 \gamma_y, \qquad \beta_1 = m - 2\beta_2 \gamma_Q.$$

This yields the baseline values $\kappa_1 = 1.655$ and $\beta_1 = 1.631$.[7]

Three structural parameters—σ, δ, and ϕ—are calibrated using long-run targets, while the takeoff date t_n is calibrated from the fit to historical data. We impose standard feasibility restrictions to preserve the model's structure. The parameter σ governs the returns to product variety in the aggregate production function. On the balanced growth path, the net entry rate satisfies $n^* = \lambda/(1-\sigma)$. We target $n^* = 1.125\%$ per year, matching the average net establishment entry rate for the U.S. over 1980–2019 from the Business Dynamics Statistics (BDS). This implies $\sigma = 1 - \lambda/n^* = 0.20$, which is close to the social return to variety of 0.25 in Iacopetta et al. (2019). Because σ is pinned down directly by the entry-rate target (n^*) through a closed-form relationship, we calibrate it separately.

We then calibrate (δ, ϕ, t_n) using a nested procedure. For each candidate takeoff date $t_n \in [1, 70]$, we choose (δ, ϕ) to minimize an objective function that penalizes deviations from the long-run targets for g^* and NQ/Y, rewards a good fit to the historical log GDP per capita path, and enforces feasibility restrictions that preserve the model's three-phase structure. The inner optimization over (δ, ϕ) is carried out with the Nelder–Mead algorithm, initialized at the solution of the exactly identified two-equation system that matches the two long-run moments. We then select the calibrated takeoff date t_n as the grid value that minimizes the SSE of log GDP per capita over 1990–2020 and retains the associated (δ, ϕ).

The calibration yields $\delta = 7.623$ and $\phi = 0.159$. The implied steady-state growth rate is $g^* = 2.43\%$ and the vertical R&D share is 1.46%. These values depart modestly from the exact targets because the calibration balances long-run discipline with a realistic transition and a good fit to the historical path, while maintaining a clean separation between the model's phases.

The takeoff date is the year in which the firm-size state variable x_t first crosses the horizontal innovation threshold x_N. Before this date, the economy is in the pre-industrial phase, with $n_t = z_t = 0$ and x_t growing at the population growth rate λ. Consistent with the nested calibration procedure above, the search over $t_n \in [1, 70]$ (corresponding to calendar years 1801–1870) selects the takeoff date that yields the lowest SSE of log GDP per capita over 1990–2020 after re-optimizing (δ, ϕ) for each grid value. The calibrated takeoff date is $t_n = 2$, corresponding to calendar year $T_n = 1802$, implying that horizontal innovation begins almost immediately at the start of the sample. This is consistent with the historical record: the first decades of the nineteenth century saw rapid growth in the number of U.S. manufacturing establishments and patents (see Sokoloff and Engerman, 2000).[8] Despite adopting a more restrictive calibration strategy than Iacopetta and Peretto (2021), we estimate a takeoff year that is very close to their imposed date ($T_n = 1800$), which follows the growth literature in general (e.g., Lucas, 2000). This emphasizes the model's ability to quantify such events endogenously.

Given t_n, three restrictions pin down the initial state (x_0, N_0, Z_0). First, the pre-industrial dynamics $x_t = x_0 \exp(\lambda t)$ and the threshold-crossing condition $x(t_n) = x_N$ imply $x_0 = x_N \exp(-\lambda t_n)$. Second, the identity $x_0 = K/N_0^{1-\sigma}$ determines the initial number of varieties. Third, initial GDP per capita from the Maddison data ($y_{1800} \approx \$2,545$ in 2011 international dollars) determines the initial quality index through $y_0 = A \cdot N_0^{\sigma} \cdot Z_0$. The implied values are $x_0 = 1.259$, $N_0 = 0.152$, and $Z_0 = 15{,}879$ (see Table 2). [9]

5.2. Baseline results

Table 3 reports the key thresholds and implied objects. The ordering $x_0 < x_N < x_Z < x^*$ is maintained throughout. The calibrated model generates a trajectory that maps onto U.S. historical experience.

Before the economy's takeoff, firm size grows at the population growth rate λ, but neither form of innovation is profitable. Per capita output is essentially constant, which is consistent with the Maddison data showing stagnant living standards before 1800. The calibrated takeoff date of 1802 places the onset of horizontal innovation at the beginning of the nineteenth century. Iacopetta and Peretto (2021) choose the activation of horizontal innovation to occur at $T_n = 1800$. Our calibrated value is very close to that date.

Once x_t crosses x_N, entry of new product varieties becomes profitable and the per capita growth rate $g_t = \sigma n_t$ begins to rise. Growth remains modest—reaching approximately 0.13% by 1825—because it comes entirely from the extensive (variety) margin. During this phase, firm size increases from $x_N = 1.281$ towards the vertical threshold $x_Z = 1.469$.

When x_t crosses x_Z, quality-improving R&D becomes active. The vertical innovation start date calibrated here ($T_z = 1825$) is relatively close to the corresponding year calibrated in Iacopetta and Peretto (2021). The robustness of the takeoff years for horizontal and vertical innovation (T_n, T_z) across different calibration strategies is attributable to the strength of the modeling framework. The growth rate accelerates sharply as the intensive margin (vertical innovation) joins the extensive margin. By 1900, g_t exceeds 1.5%, and the economy converges gradually towards the balanced growth path. The resulting concave growth acceleration matches the characteristic shape documented in the historical data.

[7] We address the arbitrariness of the ruling elite's preference parameters further in the next subsection.

[8] The fit is not sensitive to the exact takeoff date within a narrow window: any $t_n \in [1, 10]$ produces a similar SSE, because the Maddison data show essentially flat living standards before and around 1800.

[9] The two composite expressions used here are defined as $K \equiv \theta^{\frac{1}{1-\theta}}$ and $A \equiv \left(\frac{(1-\gamma_y)\theta}{\mu}\right)^{\frac{\theta}{1-\theta}}$.

Table 1
Calibrated parameters.

Parameter	Description	Value
Panel A: Externally set		
ρ	Discount rate	0.03
λ	Population growth rate	0.009
θ	Intermediate goods share	0.60
α	Elasticity of profits w.r.t. quality	0.333
μ	Markup/quality step	1.55
γ_y	Output extraction rate	0.02
γ_Q	R&D extraction rate	0.04
κ_2	Pref. param. of rul. elite	1.00
β_2	Pref. param. of rul. elite	0.80
Panel B: Internally calibrated		
σ	Returns to variety	0.20
δ	Entry cost	7.623
ϕ	Fixed operating cost	0.159
T_n	Takeoff year	1802
κ_1	Pref. param. of rul. elite	1.655
β_1	Pref. param. of rul. elite	1.631

Table 2
Targeted moments.

Moment	Target	Model	Identifies
Long-run growth rate, g^*	2.00%	2.43%	δ, ϕ
Vertical R&D share, NQ/Y	2.00%	1.46%	δ, ϕ
Net entry rate, n^*	1.125%	1.125%	σ
GDP p.c. in 1800	$2545	Matched	Z_0
GDP p.c. path, 1990–2020	Maddison	SSE = 5.54	t_n

Notes: The growth rate and R&D share targets are balanced growth path values. The model column reports the steady-state values implied by the calibrated parameters. The departure from the 2% targets reflects the trade-off in the objective function between asymptotic moments and fitting the historical transition. The entry rate n^* is matched exactly because σ is pinned down from this moment alone.

Table 3
Derived objects and phase thresholds.

Object	Value	Interpretation
x_0	1.259	Initial firm size (1800)
x_N	1.281	Horizontal innovation threshold
x_Z	1.469	Vertical innovation threshold
x^*	1.881	Long-run steady state (both margins active)
Takeoff year	1802	x_t crosses x_N
Vertical start	1825	x_t crosses x_Z
n^*	1.125%	Variety growth on BGP
z^*	2.21%	Quality growth on BGP
g^*	2.43%	Per capita growth on BGP

Fig. 4 plots the model-implied GDP per capita against the Maddison data from 1650 to 2022. The model tracks the data closely, capturing the extended period of near-stagnation, the gradual takeoff, and the sustained growth acceleration. Fig. 5 shows the implied per capita growth rate, with vertical dashed lines marking the phase transitions. Fig. 6 displays the evolution of the firm-size state variable x_t, which rises smoothly through the two innovation thresholds and converges towards x^*.

Before turning to the counterfactual experiments, we examine the sensitivity of the extraction block to the choice of ruling-elite preference parameters. A natural concern is the arbitrariness in specifying $(\kappa_1, \kappa_2, \beta_1, \beta_2)$, especially because two of these parameters are pinned down by matching target extraction rates. To address this concern, we evaluate the implied extraction-rate difference $\Delta \equiv \gamma_Q^* - \gamma_y^*$ over a broad grid of linear preference-parameter pairs $(\kappa_1, \beta_1) \in [0.5, 3] \times [0.1, 2.5]$, while varying the nonlinear preference pair (κ_2, β_2) and the political-risk threshold Ψ.

Fig. 7 presents this robustness check as a 3×3 array of heat maps. The horizontal axis records κ_1 (the elite's linear political cost of extracting output), the vertical axis records β_1 (the corresponding linear political cost of extracting R&D inputs), and the triangular region $\kappa_1 \leq \beta_1$ is excluded to impose the maintained restriction $\kappa_1 > \beta_1$. Rows correspond to low, medium, and high values of the political-risk threshold Ψ, while columns correspond to alternative nonlinear preference pairs (κ_2, β_2). The white marker in each panel denotes the reverse-calibrated pair (κ_1, β_1) that reproduces the target extraction rates $(\gamma_y, \gamma_Q) = (0.02, 0.04)$ for that panel's parameters $(\kappa_2, \beta_2, \Psi)$. Across all panels, the qualitative ordering $\gamma_Q^* > \gamma_y^*$ is preserved throughout the displayed admissible region. Hence, the central qualitative implication of the model is not driven by an arbitrary baseline choice of ruling-elite preference parameters.

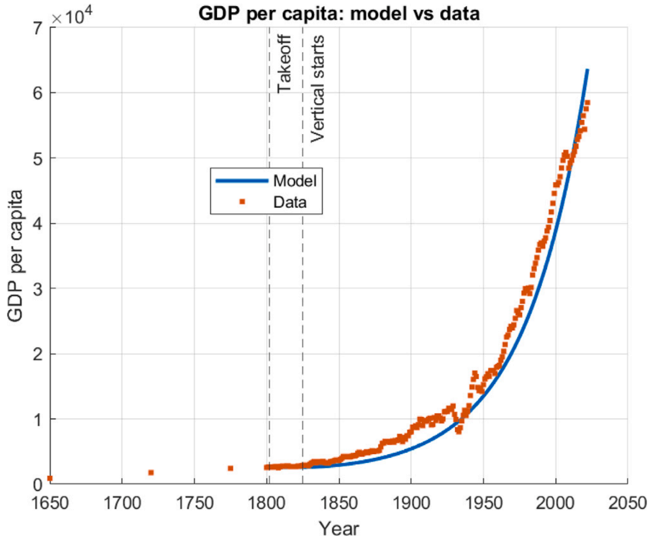

Fig. 4. GDP per capita: model vs. Maddison data.
Notes: The solid line is the model prediction; dots are the data of the Maddison Project Database (2023). Vertical dashed lines mark the estimated takeoff year (1802) and the onset of vertical innovation (1825). GDP per capita is in 2011 international dollars. The calibration chooses (δ, ϕ, t_n) to balance long-run targets with the fit to the historical transition, subject to feasibility conditions that preserve the phase structure.

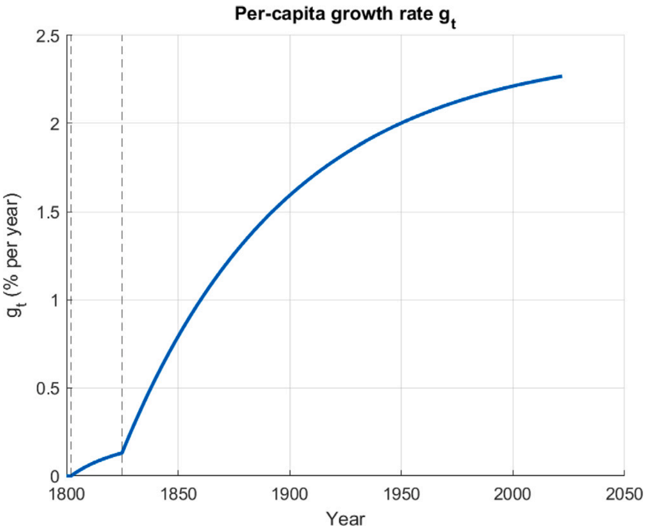

Fig. 5. Per capita growth rate.
Notes: Growth rate $g_t = \sigma n_t + z_t$ implied by the calibrated model. Vertical dashed lines mark the phase transitions.

5.3. Counterfactual experiments

We assess how extraction rates affect the growth trajectory through three complementary experiments. The first varies γ_Q and γ_y one at a time, holding all technology parameters (δ, ϕ, σ) fixed at their baseline values. This isolates the main extraction channels by showing how R&D and output extraction distort innovation incentives without altering the economy's technological capacity. The second experiment holds total extraction $B = \gamma_y + \gamma_Q$ fixed and varies the composition, directly measuring the cost of shifting

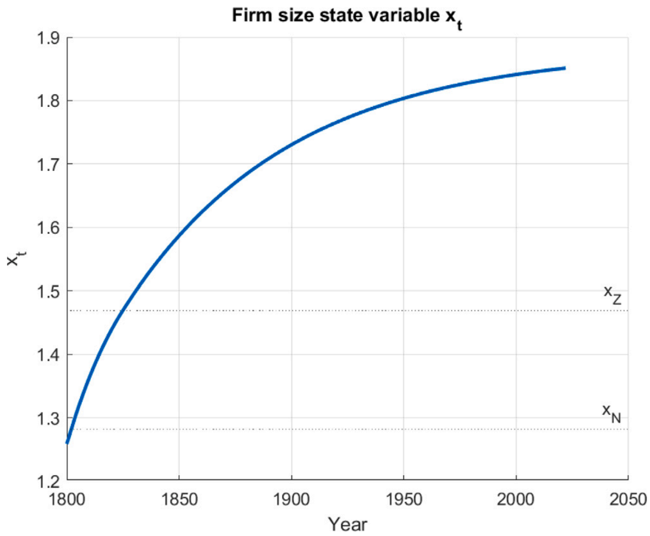

Fig. 6. Firm-size state variable x_t.
Notes: Dotted horizontal lines mark the threshold values x_N and x_Z. The state variable converges to $x^* = 1.881$ on the balanced growth path.

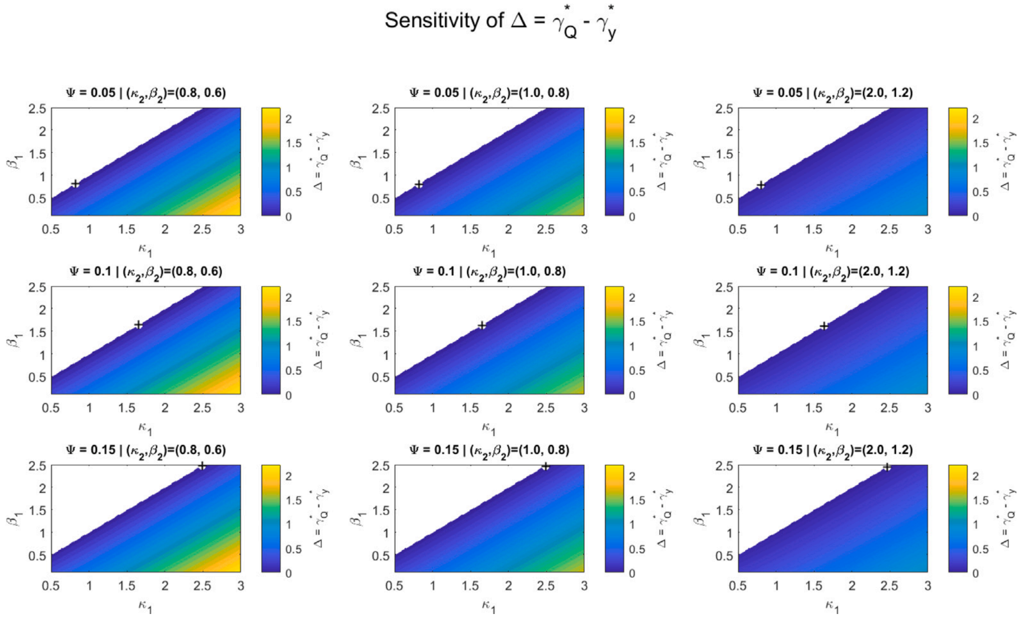

Fig. 7. Sensitivity of the extraction rates under different preferences of the ruling elite.
Notes: In each panel, we plot the heat map of $\Delta(\kappa_1, \beta_1) \equiv \gamma_Q^* - \gamma_y^*$ over $\kappa_1 \in [0.5, 3]$ and $\beta_1 \in [0.1, 2.5]$, excluding the triangular region $\kappa_1 \leq \beta_1$. Rows correspond to low, medium, and high values of the political-risk threshold $\Psi \in \{0.05, 0.10, 0.15\}$. Columns correspond to alternative nonlinear preference parameter pairs $(\kappa_2, \beta_2) \in \{(0.8, 0.6), (1.0, 0.8), (2.0, 1.2)\}$, with the center column representing the baseline pair. The dashed diagonal line indicates $\kappa_1 = \beta_1$ for reference. The white marker in each panel denotes the reverse-calibrated pair (κ_1, β_1) that reproduces the target extraction rates $(\gamma_y, \gamma_Q) = (0.02, 0.04)$ for that panel's $(\kappa_2, \beta_2, \Psi)$. Colors indicate the magnitude of Δ, with lighter colors corresponding to larger values of Δ. The color scale is held fixed across panels. In all displayed panels, $\Delta > 0$ throughout the plotted admissible region, implying $\gamma_Q^* > \gamma_y^*$ robustly across a wide range of plausible parameter choices. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 4
Counterfactual: varying quality extraction γ_Q.

γ_Q	Vertical start year	Delay vs. baseline (years)	g^* (% p.a.)
0.00	1821	−4.2	2.86
0.02	1823	−2.2	2.76
0.04*	1825	0 (baseline)	2.43
0.06	1827	+2.5	2.22
0.08	1830	+5.4	2.00
0.10	1834	+8.7	1.79

Notes: Technology parameters held fixed at baseline values. The asterisk denotes the baseline calibration. Growth rates are balanced growth path values g^*. The delay is measured relative to the baseline vertical innovation start year (1825).

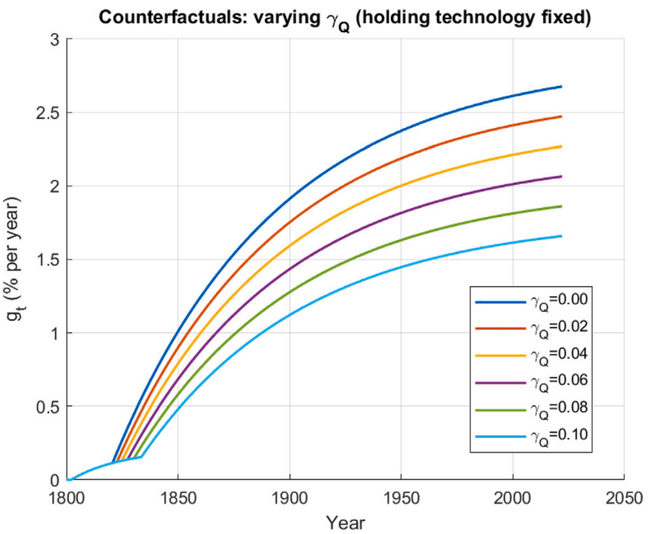

Fig. 8. Counterfactuals: varying γ_Q (holding technology fixed).
Notes: Each curve varies γ_Q while holding all other parameters at baseline values.

extraction towards R&D inputs. Finally, we assess whether observed cross-country income gaps can be accounted for by extraction rates alone, treating (γ_y, γ_Q) as the only free parameters.

5.3.1. Varying R&D extraction rates

The R&D extraction rate operates through two channels. First, it reduces the profitability of vertical R&D, delaying the activation of Phase 2. Second, it decreases the balanced growth path growth rate by lowering the return to quality investment. These effects are quantitatively substantial.

Table 4 reports the implied start year of vertical innovation and the delay relative to the baseline for $\gamma_Q \in [0.00, 0.10]$. Increasing γ_Q from the baseline value of 0.04 to 0.10 delays the onset of vertical innovation by nearly nine years—from 1825 to 1834—and reduces the long-run balanced growth rate by about 0.6 percentage points. Even moderate increases in R&D extraction have large cumulative effects: $\gamma_Q = 0.06$ delays vertical innovation by 2.5 years relative to the baseline, and the associated growth shortfall accumulates over time as the economy converges more slowly and to a lower balanced growth rate.

Fig. 8 plots the growth-rate paths. A higher γ_Q both delays the takeoff of Phase 2 (shifting curves rightward) and depresses the long-run growth rate (shifting them downward). The gap between the $\gamma_Q = 0$ and $\gamma_Q = 0.10$ paths widens over time, illustrating the compounding nature of the growth losses. By 2022, the economy with $\gamma_Q = 0.10$ has a growth rate that is around 60% of the extraction-free benchmark.

5.3.2. Varying output extraction rates

Next, we examine the growth consequences of output extraction by varying γ_y, while holding the technology parameters and initial conditions fixed at their U.S. baseline values. This exercise isolates the transitional effects of output extraction from its long-run consequences—a distinction that is central to our theoretical framework.

The results are reported in Table 5 and Fig. 9. Two features stand out. First, the balanced growth rate g^* is identical across all values of γ_y, in line with the model's predictions. Output extraction scales down static profits proportionally for incumbents and entrants, so it does not distort the long-run allocation of resources between horizontal and vertical R&D. Second, despite this long-run neutrality, output extraction has large level effects. An economy with $\gamma_y = 0.10$ reaches a GDP per capita in 2022 that is substantially

Table 5
Counterfactual: varying output extraction γ_y.

γ_y	Horizontal start year	Delay vs. baseline (years)	g^* (% p.a.)
0.00	1796	−5.6	2.43
0.01	1799	−2.2	2.43
0.02*	1802	0	2.43
0.04	1808	+5.7	2.43
0.06	1813	+11.6	2.43
0.08	1820	+17.5	2.43

Notes: Technology parameters held fixed at baseline values. The asterisk denotes the baseline calibration. Growth rates are balanced growth path values g^*. The delay is measured relative to the baseline horizontal innovation start year (1802).

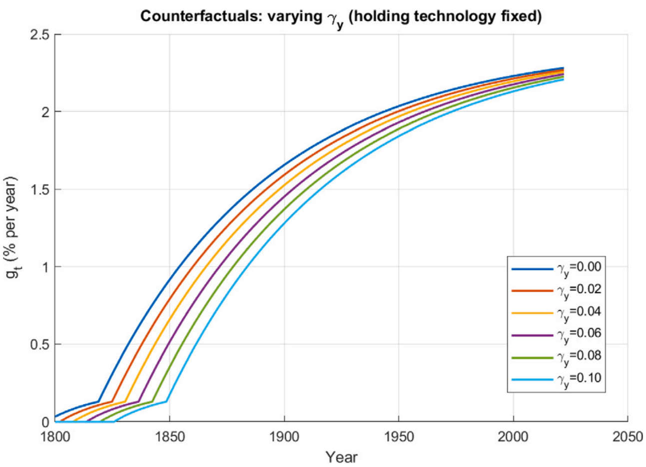

Fig. 9. Counterfactuals: varying γ_y (holding technology fixed).
Notes: Each curve varies γ_y while holding all other parameters at baseline values.

below the baseline ($\gamma_y = 0.02$), whereas an economy with $\gamma_y = 0$ is correspondingly richer. This occurs through several channels: higher output extraction (i) directly reduces productivity, (ii) raises the takeoff threshold x_N, delaying industrialization, and (iii) slows the transition dynamics in both phases by reducing innovation rates for any given firm size x_t. Despite these quantitatively large level effects, g^* remains invariant to γ_y because these channels cancel each other in the steady-state growth expression. Income differences generated by output extraction therefore reflect foregone levels—not permanently lower growth—in contrast to the compounding effects of R&D extraction documented above.

5.3.3. *The composition of extraction*

The one-at-a-time counterfactuals change total extraction as well as its composition. To isolate the composition channel, we hold the total extraction budget $B \equiv \gamma_y + \gamma_Q$ fixed and redistribute extraction between output and R&D. Total resources extracted by the ruling elite are therefore unchanged; only the margin from which extraction is taken differs.

We begin with $B = 0.06$, the baseline sum. Table 6 reports seven compositions ranging from $(\gamma_y, \gamma_Q) = (0.06, 0)$ (all output extraction) to $(0, 0.06)$ (all R&D extraction). Shifting extraction entirely from output to R&D reduces GDP per capita in 2022 from \$83,585 to \$54,013—a ratio of 0.65—and lowers the balanced growth rate from 2.86% to 2.22%. Thus, holding total extraction fixed, concentrating extraction on R&D yields roughly two-thirds of the 2022 income attained under pure output extraction. At the baseline composition $(\gamma_y, \gamma_Q) = (0.02, 0.04)$, GDP per capita is \$63,661, which is 76% of the all-output-extraction benchmark. Even within the plausible range of U.S. extraction rates, the composition matters quantitatively.

Figs. 10 and 11 display the GDP per capita paths and growth rates, respectively. On the log-GDP plot, the paths share a common origin and are essentially indistinguishable through about 1830, but they progressively separate as differences in long-run growth compound. By 2022, the gap between the extreme compositions is about 0.44 log-points, corresponding to roughly a 1.55-fold income difference.

Table 7 extends the analysis to four total extraction budgets $B \in \{0.04, 0.06, 0.08, 0.10\}$. For each budget, we report GDP per capita in 2022 and the associated income ratio at five compositions $\gamma_Q/B \in \{0, 0.25, 0.50, 0.75, 1.0\}$. Two regularities stand out. First, within each budget level, the income ratio is monotonically decreasing in the R&D share of extraction: shifting extraction towards R&D lowers income. Second, the income cost of shifting extraction towards R&D is larger at higher total extraction. At $B = 0.04$, moving from $\gamma_Q/B = 0$ to $\gamma_Q/B = 1$ lowers the income ratio from 1.00 to 0.745 (a 25.5% loss). At $B = 0.10$, the same shift lowers the ratio from 1.00 to 0.498 (a 50.2% loss).

Table 6
Varying the composition of extraction at $B = \gamma_y + \gamma_Q = 0.06$.

γ_y	γ_Q	GDP p.c. 2022	Ratio	g^* (%)
0.06	0.00	83,585	1.000	2.86
0.05	0.01	78,770	0.942	2.76
0.04	0.02	73,795	0.883	2.65
0.03	0.03	68,735	0.822	2.54
0.02*	0.04*	63,661	0.761	2.43
0.01	0.05	58,564	0.702	2.33
0.00	0.06	54,013	0.646	2.22

Notes: Total extraction $B = 0.06$ is held fixed. Ratio denotes GDP per capita relative to the all-output-extraction case ($\gamma_Q = 0$). The asterisk denotes the baseline calibration. Growth rates are balanced growth path values.

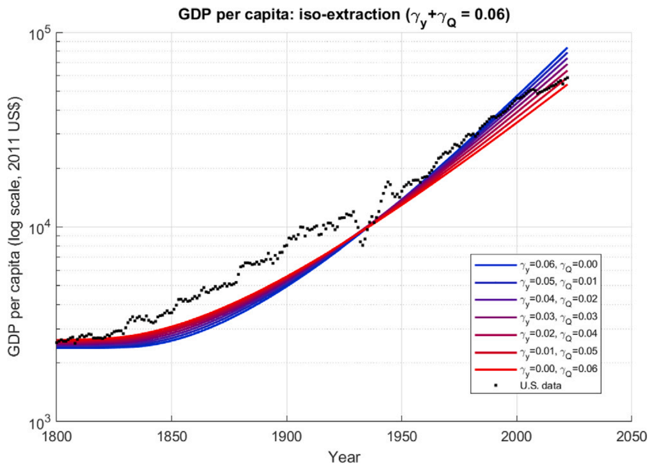

Fig. 10. GDP per capita under extraction ($B = 0.06$, log scale).
Notes: Each path holds $\gamma_y + \gamma_Q = 0.06$ fixed and varies the composition. Colors range from blue (all output extraction) to red (all R&D extraction). Dots show U.S. Maddison data. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 12 summarizes this pattern. Each curve traces the income ratio against the R&D share of extraction for a given budget. The curves are downward sloping and increasingly steep at higher budget levels. In this calibration, for an economy with total extraction of 10%, shifting extraction from output towards R&D reduces 2022 income to roughly half of its level under pure output extraction.

Two features of Table 7 deserve emphasis. First, the balanced growth rate in the $\gamma_Q = 0$ cases is constant at the technology-determined ceiling (here 2.86%), consistent with Proposition 4. Output extraction γ_y affects thresholds and transition dynamics and can therefore affect income levels over finite horizons. In this calibration, these level effects are comparatively small relative to changes in γ_Q. Second, holding γ_y fixed, the balanced growth rate declines monotonically as γ_Q increases. Taken together, these results show that the composition of extraction can be as important as—or more important than—the total amount. For example, GDP per capita in 2022 under $(B = 0.10, \gamma_Q = 0)$ is $57,020, comparable to $61,139 under $(B = 0.06, \gamma_Q = 0.045)$, whereas concentrating a modest budget of $B = 0.04$ entirely on R&D extraction lowers the income ratio to 0.745. The policy implication is clear: for long-run development, it matters not only how much the ruling elite extracts, but from which sources.

5.3.4. Decomposing institutional sources of long-run income gaps

We proceed by assessing whether the model can rationalize observed income gaps between (groups of) countries using extraction rates as the only free parameters. We compare the U.S. to two cases: a Latin American average and the former USSR.[10] In each case, we hold all technology parameters $(\delta, \phi, \sigma, \alpha, \theta, \mu)$ at their U.S.-calibrated values and use the economy's initial GDP per capita from the Maddison data as the starting condition for comparison. We then search over (γ_y, γ_Q) to minimize the sum of squared errors in

[10] For the former USSR, continuous Maddison data begin in 1860. We impute $y_{1800} = y_{1860}$, reflecting near-zero per capita growth in the pre-industrial era. The SSE is computed only over the years for which data exist.

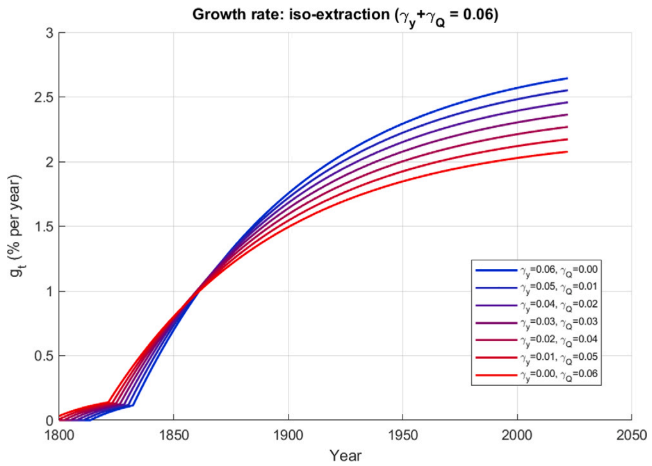

Fig. 11. Growth rate under extraction ($B = 0.06$).
Notes: Per capita growth rate g_t for each composition. The curves fan out after horizontal and vertical innovation become active. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 7
Extraction: composition effects across budgets.

B	γ_Q/B	γ_y	γ_Q	GDP p.c. 2022	Ratio	g^* (%)
0.04	0.00	0.040	0.000	100,746	1.000	2.86
	0.25	0.030	0.010	94,247	0.935	2.76
	0.50	0.020	0.020	87,657	0.870	2.65
	0.75	0.010	0.030	81,090	0.805	2.54
	1.00	0.000	0.040	75,005	0.745	2.43
0.06	0.00	0.060	0.000	83,585	1.000	2.86
	0.25	0.045	0.015	76,298	0.912	2.70
	0.50	0.030	0.030	68,735	0.822	2.54
	0.75	0.015	0.045	61,139	0.731	2.38
	1.00	0.000	0.060	54,013	0.646	2.22
0.08	0.00	0.080	0.000	69,141	1.000	2.86
	0.25	0.060	0.020	61,955	0.896	2.65
	0.50	0.040	0.040	54,222	0.784	2.43
	0.75	0.020	0.060	46,400	0.671	2.22
	1.00	0.000	0.080	39,058	0.565	2.00
0.10	0.00	0.100	0.000	57,020	1.000	2.86
	0.25	0.075	0.025	50,463	0.885	2.60
	0.50	0.050	0.050	43,035	0.755	2.33
	0.75	0.025	0.075	35,443	0.621	2.06
	1.00	0.000	0.100	28,383	0.498	1.79

Notes: Each panel holds total extraction $B = \gamma_y + \gamma_Q$ fixed at the indicated value. Ratio denotes GDP per capita relative to the all-output-extraction case ($\gamma_Q = 0$) within each budget level. Growth rates are balanced growth path values g^*. The balanced growth path growth rate in the $\gamma_Q = 0$ rows is identical across budgets because γ_y does not affect g^* (Proposition 4). Output extraction can still affect transition dynamics and thus income levels at finite horizons.

log GDP per capita between the model and the data. Thus, the model must account for up to two centuries of divergent growth paths with only two institutional wedges, while all technological differences are shut down by construction.[11]

[11] For each comparison economy (ce), the exercise proceeds in three steps: (i) set the initial condition y_0^{ce} to the economy's GDP per capita in 1800 from the Maddison database (or the imputed value for the former USSR), (ii) conduct a two-stage grid search over $(\gamma_y, \gamma_Q) \in [0, 0.25]^2$, and (iii) decompose the income gap between the counterfactual U.S.-institutions scenario and the best-fit scenario using Shapley values: $\phi_{\gamma_y} = \frac{1}{2}\left[(\ln y_U - \ln y_Y) + (\ln y_Q - \ln y_B)\right]; \phi_{\gamma_Q} = \frac{1}{2}\left[(\ln y_U - \ln y_Q) + (\ln y_Y - \ln y_B)\right]$. Here, y_U is terminal GDP per capita under U.S. institutions $(\gamma_y^{US}, \gamma_Q^{US})$ with the comparison economy's initial condition, y_Y replaces only γ_y with the calibrated value γ_y^*, y_Q replaces only γ_Q with γ_Q^*, and y_B replaces both (the best fit). The Shapley values sum exactly to the total log-income gap $\ln y_U - \ln y_B$ and distribute any interaction term symmetrically across both orderings (Shorrocks, 2013).

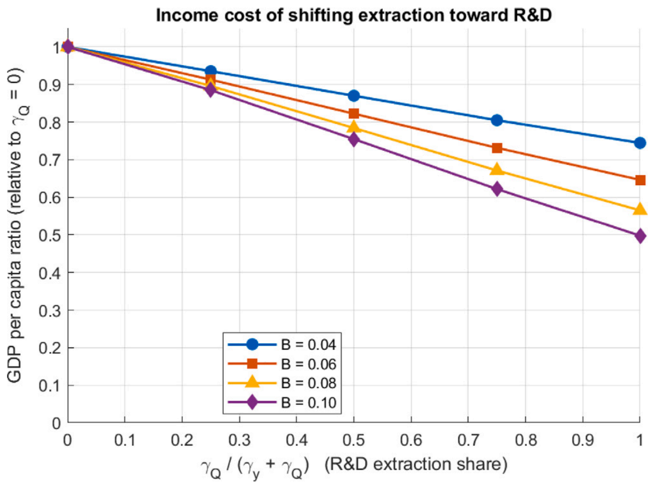

Fig. 12. Income cost of shifting extraction towards R&D.
Notes: Each curve plots the GDP per capita ratio (relative to $\gamma_Q = 0$) against the R&D share of total extraction γ_Q/B for a given budget level B. All curves start at 1.0 (all output extraction) and decline monotonically.

We construct a representative Latin American economy as the simple (unweighted) average of GDP per capita across Argentina, Brazil, Chile, Colombia, Mexico, and Peru—six major economies with continuous Maddison data from 1800 onward. Averaging across countries smooths idiosyncratic shocks that a deterministic model cannot capture.

The average initial income is $y_{1800} = \$1,085$, roughly 43% of the U.S. level. By 2022, the average has risen to $\$16,523$, approximately 28% of U.S. GDP per capita, indicating persistent divergence. The grid search yields $\gamma_y^* = 0.00$ and $\gamma_Q^* = 0.076$. Thus, the best fit assigns essentially no role to output extraction and attributes divergence to R&D extraction, with the implied total extraction $B^* = 0.076$ falling within the range calibrated by Chu et al. (2022) for developing economies. The fit is remarkable: SSE = 0.63 in log GDP per capita, and the model predicts $\$17,224$ in 2022, very close to the observed average.

Table 8 reports the decomposition. Under U.S. institutions, Latin America would reach a GDP per capita of $\$27,131$ by 2022. The total log-income gap between U.S. institutions and the best fit is $\ln(27,131) - \ln(17,224) = 0.454$. The Shapley decomposition reveals an important asymmetry. R&D extraction explains 0.577 log points (127% of the gap), while output extraction contributes −0.123 log points (−27%). The negative contribution reflects interaction effects: replacing only γ_y while holding γ_Q fixed at its estimated level raises income relative to the U.S. baseline configuration. In other words, the entire gap—and more—is therefore driven by γ_Q. Output extraction does not reduce long-run growth (consistent with Proposition 4) and, here, partially offsets the level effects of R&D extraction.

Fig. 13 shows that the model closely tracks the Latin American trajectory. The dashed blue counterfactual (U.S. institutions) lies persistently above the red best-fit path, with the gap widening gradually over time. Fig. 14 shows the growth dynamics: under U.S. institutions the economy converges towards $g^* = 2.434\%$, whereas under the calibrated Latin American institutions it converges to $g^* = 2.047\%$. The lower long-run growth rate is entirely driven by γ_Q.

The Soviet case provides a contrast. Maddison data for the former USSR (code SUN) begin in 1860 at $y_{1860} = \$1,525$. The grid search yields $\gamma_y^* = 0.032$ and $\gamma_Q^* = 0.08$—higher extraction on both margins than the Latin American case. The implied total extraction is $B^* = 0.112$. The R&D extraction rate of 8% is the highest in our sample, consistent with a pattern of directing research resources towards state priorities rather than market-driven creative destruction. The fit is weaker than in the Latin American case (SSE = 26.31), reflecting the volatile historical path. Nevertheless, the model's terminal prediction $(19,095)$ is close to the observed $19,355$.

Under U.S. institutions, the former USSR would reach a per capita GDP of $38,147$ by 2022. The total log-income gap equals $\ln(38,147) - \ln(19,095) = 0.692$. The Shapley decomposition assigns 0.621 log points (89.8%) to R&D extraction and 0.071 log points (10.2%) to output extraction. Thus, unlike the Latin American case, γ_y plays a modest but positive role in explaining Soviet divergence. Still, the overwhelming share of the gap is due to γ_Q.

Fig. 15 shows that the model captures the long-run trend in Soviet income despite missing short-run volatility. Fig. 16 illustrates that long-run growth under the best fit converges to $g^* = 2.004\%$, compared to 2.434% under U.S. institutions. As in the Latin American case, the persistent growth differential arises from R&D extraction.

Three features of these results merit comment. First, in both cases the dominant mechanism behind divergence is R&D extraction. Because γ_Q directly reduces vertical innovation, it lowers the balanced growth path growth rate and generates permanently lower growth. Output extraction, by contrast, affects thresholds and transition dynamics but does not alter g^*. Second, the quantitative contribution of γ_y differs across cases. In Latin America, it partially offsets the effect of γ_Q, yielding a negative Shapley share. In the Soviet case, it contributes positively but its importance remains secondary by a large margin. Thus, while composition dominates amount, the interaction between the extraction rates matters for level effects. Third, the exercise is strongly over-identified: hundreds

Table 8
Great Divergence: estimated extraction rates and income decomposition.

	Latin America	Former USSR
Panel A: Data		
y_{1800} (2011 int. $)	1085	1525[a]
y_{2022} (2011 int. $)	16,523	19,355
y_{2022}^{US} (2011 int. $)	58,487	
Panel B: Estimated extraction rates		
γ_y^*	0.000	0.032
γ_Q^*	0.076	0.080
$B^* = \gamma_y^* + \gamma_Q^*$	0.076	0.112
SSE (log GDP p.c.)	0.63	26.31
Panel C: Model predictions (2022, in 2011 int. $)		
y_U: U.S. institutions, comparison y_0	27,131	38,147
y_Y: only γ_y at est. level	31,011	35,302
y_Q: only γ_Q at est. level	15,399	20,349
y_B: both at est. level (best fit)	17,224	19,095
Panel D: Balanced growth rates (%)		
g^* under U.S. institutions	2.43	2.43
g^* under best fit	2.05	2.00
Panel E: Shapley decomposition		
Total gap ($\ln y_U - \ln y_B$)	0.454	0.692
ϕ_{γ_y} (output extr.)	−0.123 (−27%)	0.071 (10.2%)
ϕ_{γ_Q} (R&D extr.)	0.577 (127%)	0.621 (89.8%)

Notes: Technology parameters are held at U.S.-calibrated values. Latin America is the unweighted average of Argentina, Brazil, Chile, Colombia, Mexico, and Peru. y_U, y_Y, y_Q, y_B are evaluated at the comparison economy's initial condition y_0 with the indicated extraction rates. Percentages in Panel E are shares of the total gap.
[a] Imputed from the 1860 value; Maddison data for the former USSR begin in 1860.

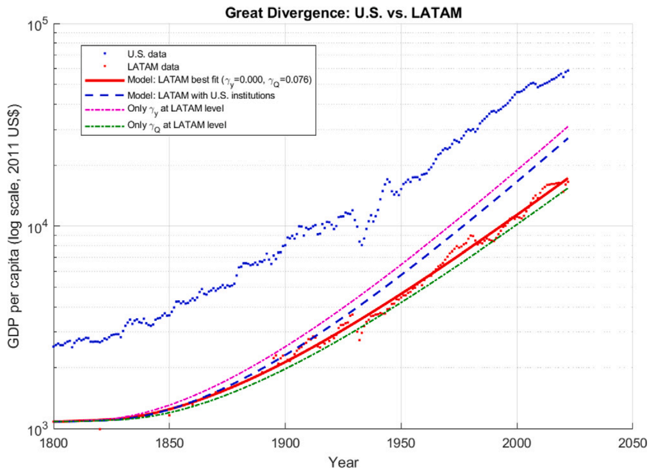

Fig. 13. Great divergence: U.S. vs. Latin America.
Notes: Dots show Maddison data (U.S. in blue, Latin American average in red). Solid red: best-fit model path ($\gamma_y^* = 0.000, \gamma_Q^* = 0.076$). Dashed blue: counterfactual with U.S. institutions applied to Latin American initial conditions. Log scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

of annual data points are matched with only two free parameters. The Latin American fit is remarkably tight; the Soviet fit is looser but economically informative. The calibrated R&D extraction rates—7.6% for Latin America and 8.0% for the former USSR—are economically plausible and consistent with the model's central mechanism. Taken together, these counterfactual exercises confirm the results of the previous experiment: what matters most for long-run development is not how much elites extract, but whether extraction distorts the innovation margin.

6. Conclusion

This paper develops a dynamic Schumpeterian growth model to analyze how extractive institutions in the form of the resource appropriation by a politically dominant elite shape the transition from Malthusian stagnation to sustained long-run economic growth.

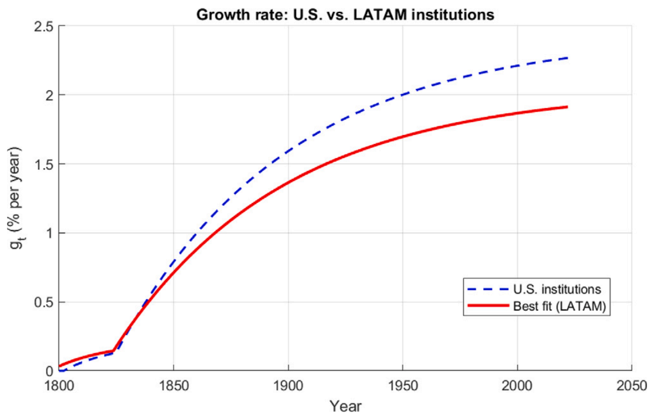

Fig. 14. Growth rate: U.S. vs. Latin American institutions.
Notes: Dashed: growth rate path under U.S. institutions with Latin American initial conditions. Solid: best-fit path.

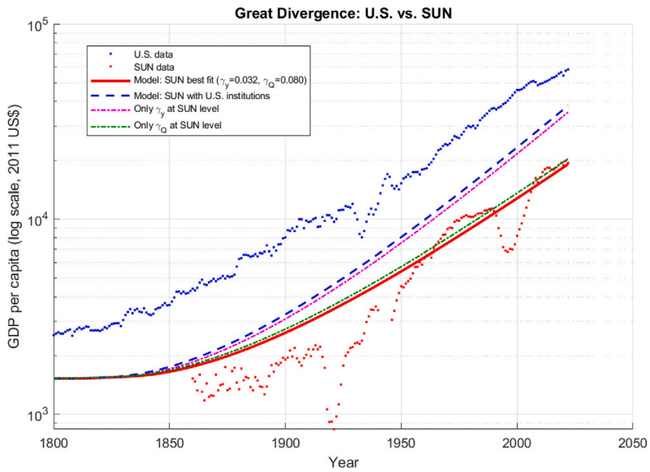

Fig. 15. Great divergence: U.S. vs. former USSR.
Notes: Dots show Maddison data (U.S. in blue, former USSR in red). Solid red: best-fit model path ($\gamma_y^* = 0.032, \gamma_Q^* = 0.080$). Dashed blue: counterfactual with U.S. institutions applied to Soviet initial conditions. Log scale. Data for the former USSR begin in 1860. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

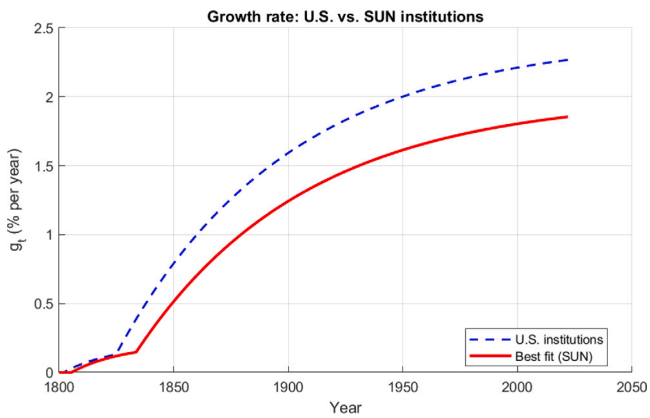

Fig. 16. Growth rate: U.S. vs. Soviet institutions.
Notes: Dashed: growth rate path under U.S. institutions with Soviet initial conditions. Solid: best-fit path.

We allow for two dimensions of extraction, from total output in the economy and from resources devoted to R&D that leads to creative destruction. The ruling elite then chooses the extraction rates considering their preferences and a political constraint that allows them to stay in power. The resulting framework allows us to study how the behavior of the elite, representing the extractiveness of institutions, affects both the timing of the industrial takeoff and the economic growth paths throughout the different stages of development.

Our results demonstrate that the extraction from final output delays the onset of industrialization and reduces the growth rate in the transition period to the second phase of industrialization when vertical innovation emerges as the key engine of technological progress and economic growth. Overall, however, the extraction from output leaves long-run economic growth at the balanced growth path unchanged. By contrast, extraction from R&D leaves the timing of the takeoff unchanged, but has crucial effects not only on economic growth during the second phase of industrialization but also on the long-run balanced growth path. The distinction between the two sources from which the ruling elite extracts its resources has crucial implications for how institutional settings can shape long-run economic development patterns. We calibrate the model to the U.S. economy and show that our qualitative results hold under plausible parameter constellations and that they are robust towards a wide range of different scenarios with different parameter settings.

Overall, our results show the crucial importance of institutional quality in the economic growth process (Acemoglu and Robinson, 2012; Mokyr, 2016). Extraction by the ruling elite can hamper economic development and delay the takeoff severely. In addition, extractive elites can even affect the long-run economic growth rate when they target innovation. From the perspective of development, it would therefore be of utmost importance to restrict the extractive capacities of the ruling elite, for example, by holding them accountable so that their survival constraint does not allow them to extract vast amounts of resources.

In the modern context, extractive elites may receive rents in many resource-rich economies, which amounts to output extraction. Innovation extraction, in turn, could come in the form of limiting the introduction of digital innovation or the access to information in many autocracies. From the perspective of fostering economic development, it would be highly important to restrict the ability of the elite to pursue such extractive policies.

In terms of future research, it would be interesting to integrate modern-day corruption and anti-corruption measures (e.g., along the lines of Aghion et al., 2016; Gustaffsson et al., 2025) into such a model of the historical takeoff to economic growth under extractive institutions. In such a setting, corruption would likely have more adverse effects in economies with extractive institutions.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Martin Stojanovikj reports financial support was provided by the Department of Science, Universities and Innovation of the Basque Government (IT1429-22 and IT1793-26). If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

The dynamic optimization problem in Section 2.3 can be solved by defining a current-value Hamiltonian $J_t(i)$, where monopolistic firms choose the price $P_t(i)$ and the level of in-house R&D $Q_t(i)$, while $Z_t(i)$ is the state variable and $\upsilon_t(i)$ and $\epsilon_t(i)$ are the co-state variables:

$$J_t(i) = [\pi_t(i) - Q_t(i)] + \upsilon_t(i)[(1-\gamma_Q)Q_t(i)] + \epsilon_t(i)[\mu - P_t(i)]. \tag{A.1}$$

The first-order conditions of this problem are

$$\frac{\partial J_t(i)}{\partial P_t(i)} = 0 \quad \Rightarrow \quad \frac{\partial \pi_t(i)}{\partial P_t(i)} = \epsilon_t(i), \tag{A.2}$$

$$\frac{\partial J_t(i)}{\partial Q_t(i)} = 0 \quad \Rightarrow \quad \upsilon_t(i) = \frac{1}{1-\gamma_Q}, \tag{A.3}$$

$$r_t\upsilon_t(i) - \dot{\upsilon}_t(i) = \frac{\partial J_t(i)}{\partial Z_t(i)}. \tag{A.4}$$

Given (A.3), replacing the co-state variable in (A.4) and solving the partial derivative with respect to the state variable $Z_t(i)$ yields

$$r_t = (1-\gamma_Q)\left\{\alpha\left[(P_t(i)-1)\left(\frac{(1-\gamma_y)\theta}{P_t(i)}\right)^{\frac{1}{1-\theta}}\frac{L_t}{N_t^{1-\sigma}} - \phi\right]\frac{Z_t^{1-\alpha}}{Z_t(i)^{1-\alpha}}\right\}. \tag{A.5}$$

If the price constraint is not binding, i.e., $\mu > P_t(i)$, then $\epsilon_t(i) = 0$, and $\frac{\partial \pi_t(i)}{P_t(i)} = 0$. It follows that $P_t(i) = \frac{1}{\theta}$, which is the price if the incumbent engages in Bertrand pricing competition. When $\mu \le P_t(i)$, the price constraint is binding, and the price is $P_t(i) = \mu$. Furthermore, this implies that $\mu < \frac{1}{\theta}$.

Appendix B

Here, we show the dynamics of the (worker) consumption-to-output ratio $\frac{c_t^w}{y_t}$. We start with the equilibrium condition

$$a_t = \frac{V_t N_t}{L_t}. \tag{B.1}$$

Using the free entry condition in (28) and (17) yields

$$a_t = \frac{\delta X_t N_t}{L_t} = \frac{\delta(1-\gamma_y)\theta Y_t}{\mu L_t} = \frac{\delta}{\mu}(1-\gamma_y)\theta y_t, \tag{B.2}$$

which establishes a relationship between assets and output per capita. Differentiating (B.2) with respect to time, and using (3) and (15) yields

$$\dot{y}_t = (r_t - \lambda) y_t + \frac{(1-\theta)\mu}{\theta\delta} y_t - \frac{\mu}{\theta\delta(1-\gamma_y)} c_t^w. \tag{B.3}$$

Dividing both sides of (B.3) by y_t and substituting (4) yields

$$\frac{\dot{c}_t^w}{c_t^w} - \frac{\dot{y}_t}{y_t} = \frac{\mu}{\theta\delta(1-\gamma_y)} \frac{c_t^w}{y_t} - \left(\frac{(1-\theta)\mu}{\theta\delta} + \rho - \lambda \right). \tag{B.4}$$

Therefore, (B.4) shows that the dynamics of the (worker) consumption-to-output ratio c_t^w/y_t is characterized by saddle-point stability, and c_t^w/y_t must jump to its steady-state value

$$\frac{c_t^w}{y_t} = (1-\gamma_y)\left[1 - \theta + \frac{\theta\delta(\rho+\lambda)}{\mu}\right] \tag{B.5}$$

whenever the free-entry condition holds.

Appendix C

We present the dynamics of the economy when horizontal innovation takes precedence over vertical innovation in the industrial era. This implies that $n_t > 0$ and $z_t = 0$ in the first phase of the industrial era. In this phase, the rate of return on horizontal innovation is the prevailing interest rate in the economy, as $z_t = 0$, ($r_t^e = r_t$). Using the (worker) consumption-to-output ratio expression from (4) in (B.5), $r_t = \rho + g_t = \rho + \sigma n_t$, and substituting this expression and (24) into (29), holding $z_t = 0$, yields

$$n_t = \frac{1}{\delta}\left[\mu - 1 - \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{\phi}{x_t}\right] + \lambda - \rho. \tag{C.1}$$

Setting expression (C.1) to zero, we get

$$x_N = \left[\frac{\mu}{1-\gamma_y}\right]^{\frac{1}{1-\theta}} \left[\frac{\phi}{\mu - 1 - \delta(\rho - \lambda)}\right] > x_0, \tag{C.2}$$

and the growth rate of the state variable x_t can be obtained from substituting (24) back into (C.1) as

$$\dot{x}_t = \frac{1}{\delta}\left[(1-\sigma)\left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \phi - [(1-\sigma)(\mu - 1 - \delta\rho) - \delta\rho\lambda] x_t\right] > 0. \tag{C.3}$$

In the second phase of the industrial era, firms also engage in vertical innovation, implying that $n_t > 0$ and $z_t > 0$. Therefore, $r_t^q = r_t = \rho + g_t$, and from (26):

$$g_t = (1-\gamma_Q)\alpha\left[(\mu - 1)\left(\frac{1-\gamma_y}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right] - \rho > 0. \tag{C.4}$$

As both the horizontal and vertical innovation growth engines are active, substituting (C.4) in $g_t = z_t + \sigma n_t$ yields the expression for z_t

$$z_t = (1-\gamma_Q)\alpha\left[(\mu - 1)\left(\frac{1-\gamma_y}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right] - \rho - \sigma n_t > 0. \tag{C.5}$$

To find the growth rate of varieties (n_t), we substitute (24) in (29) and use $\rho + g_t = r_t^e \equiv r_t = \rho + \sigma n_t + z_t$, to get

$$n_t = \frac{1}{\delta}\left[\mu - 1 - \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{\phi(1-\gamma_Q) + z_t}{(1-\gamma_Q)x_t}\right] - \rho + \lambda > 0 \tag{C.6}$$

To find the evolution of the state variable, we start from $\frac{\dot{x}_t}{x_t} = \lambda - (1-\sigma)n_t$. Then, we use (C.5) and (C.6) to solve for n_t as a function of x_t. As such, we linearize the dynamics of x_t with respect to n_t around its steady state. Thus, the linearized dynamics of x_t can be derived as

$$\dot{x}_t = \frac{1-\sigma}{\delta}\left\{\left[(1-\alpha)\phi - \left(\frac{\rho(1-\sigma)+\sigma\lambda}{(1-\gamma_Q)(1-\sigma)}\right)\right]\left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} - \left[(1-\alpha)(\mu-1) - \delta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right]x_t\right\}, \tag{C.7}$$

which is stable, given (25). Using (C.5) and (C.6), we can express the quality growth rate z_t as a function of x_t. Setting this expression to zero yields a threshold x_z

$$x_Z \equiv \arg\underset{x}{\text{solve}}\left\{\frac{\alpha(1-\gamma_Q)(\mu-1)}{\left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}}}x + \frac{\sigma\phi\left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}}}{\delta x} = \alpha\phi(1-\gamma_Q) + \rho + \frac{\sigma(\mu-1)}{\delta} - \sigma(\rho-\lambda)\right\} > x_N \tag{C.8}$$

that ensures $z_t > 0$.

In the long run, firm size is constant, so $\frac{\dot{x}_t}{x_t} = \lambda - (1-\sigma)n_t = 0$. It follows that the steady state growth rate of varieties is

$$n^* = \frac{\lambda}{1-\sigma}. \tag{C.9}$$

Using (C.7), or substituting (C.5) in (C.6), solving for n_t as a function of x_t, and then substituting for (C.9) as the steady state value of n_t, yields the steady state firm size

$$x^* = \left(\frac{\mu}{1-\gamma_y}\right)^{\frac{1}{1-\theta}} \frac{(1-\alpha)\phi - \left[\frac{\rho(1-\sigma)+\lambda\sigma}{(1-\gamma_Q)(1-\sigma)}\right]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \frac{\sigma\lambda}{1-\sigma}\right]} > x_Z. \tag{C.10}$$

Finally, substituting (C.10) in (C.4), yields the steady state growth rate of output per capita

$$g^* = (1-\gamma_Q)\alpha\left[(\mu-1)\frac{(1-\alpha)\phi - \left[\frac{\rho(1-\sigma)+\lambda\sigma}{(1-\gamma_Q)(1-\sigma)}\right]}{(1-\alpha)(\mu-1) - \delta\left[\rho + \frac{\sigma\lambda}{1-\sigma}\right]} - \phi\right] - \rho > 0. \tag{C.11}$$

Appendix D

Proof of Proposition 2

Proof. In the pre-industrial era, from Eqs. (32) and (33) it immediately follows that $g_t = 0$ and $\frac{\dot{x}_t}{x_t} = \lambda$, so γ_y and γ_Q do not affect the dynamics in this era.

First phase of the industrial era

Taking the first derivative of (C.2) with respect to γ_y yields

$$\frac{\partial x_N}{\partial \gamma_y} = \frac{x_N}{(1-\theta)(1-\gamma_y)} > 0. \tag{D.1}$$

Thus, a higher γ_y increases the necessary level of firm size needed to start the industrial era. Since firm size grows at a constant rate, a higher γ_y delays the start of the takeoff of the industrial era of the economy.

Since in the first phase of the industrial era $z_t = 0$, it follows that $g_t = \sigma n_t$. Taking the first derivative of Eq. (35) with respect to γ_y yields

$$\frac{\partial n_t}{\partial \gamma_y} = -\frac{1}{\delta(1-\theta)}\mu^{\frac{1}{1-\theta}}\frac{\phi}{x_t}\frac{1}{(1-\gamma_y)^{\frac{2-\theta}{1-\theta}}} < 0, \tag{D.2}$$

which directly implies that a higher γ_y decreases the growth rate in the first phase of the industrial era.

From (C.2) it immediately follows that $\frac{\partial x_N}{\partial \gamma_Q} = 0$. In addition, taking the first derivative of Eq. (37) with respect to γ_Q yields $\frac{\partial g_t}{\partial \gamma_Q} = 0$. This implies that γ_Q produces no growth effects in the first phase of the industrial era.

Second phase of the industrial era

In the second phase of the industrial era, taking the derivative of Eq. (39) with respect to γ_y yields

$$\frac{\partial g_t}{\partial \gamma_y} = -\frac{\alpha(1-\gamma_Q)}{1-\theta}\frac{\mu-1}{\mu^{\frac{1}{1-\theta}}}x_t(1-\gamma_y)^{\frac{\theta}{1-\theta}} < 0. \tag{D.3}$$

Thus, the growth rate decreases with increasing γ_y. Taking the derivative of Eq. (39) with respect to γ_Q yields

$$\frac{\partial g_t}{\partial \gamma_Q} = -\alpha\left[(\mu-1)\left(\frac{1-\gamma_y}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right] < 0, \tag{D.4}$$

where the growth rate also decreases with increasing γ_Q. Taking the total derivative of Eq. (39) with respect to γ_Q and γ_y yields

$$dg_t = -\left[\alpha\left((\mu-1)\left(\frac{1}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right)d\gamma_Q + \left(\frac{\alpha}{1-\theta}\frac{\mu-1}{\mu^{\frac{1}{1-\theta}}}x_t(1-\gamma_y)^{\frac{\theta}{1-\theta}}\right)d\gamma_y\right], \tag{D.5}$$

where simultaneous infinitesimally small changes in γ_Q and γ_y produce negative effects on the growth rate in the second phase of the industrial era.

Balanced growth path

From Eq. (44) it directly follows that $\frac{\partial g^*}{\partial \gamma_y} = 0$, so the long-run growth rate of output per capita is invariant to changes in γ_y. To evaluate the long-run growth effects of γ_Q, we take the first derivative of Eq. (44) with respect to γ_Q. To simplify the derivation of Eq. (44), we can rewrite it as follows

$$g^* = \frac{\alpha A\phi}{A-C}(1-\gamma_Q) - \frac{\alpha B(1-\gamma_Q)}{(1-\sigma)A - C(1-\gamma_Q)}, \tag{D.6}$$

where $A = (\mu-1)(1-\alpha)$, $B = (\mu-1)[\rho(1-\sigma)+\lambda\sigma]$, and $C = \delta[\rho + \frac{\lambda\sigma}{1-\sigma}]$. Taking the derivative of Eq. (D.6) with respect to γ_Q yields

$$\frac{\partial g^*}{\partial \gamma_Q} = -\frac{\alpha A\phi}{A-C} - \frac{\alpha(1-\sigma)AB}{[(1-\sigma)A - C(1-\gamma_Q)]^2} < 0, \tag{D.7}$$

where the expression in (D.7) is negative, given that $A - C > 0$ follows from (39) and the rest of the terms in the expression are positive. □

Proof of Proposition 3

Proof. We set $\gamma_y = \gamma_Q = \gamma$. The pre-industrial era follows exactly the same dynamic behavior as in the general case. In the first phase of the industrial era, (D.1), (D.2), $\frac{\partial x_N}{\partial \gamma_Q} = 0$, and $\frac{\partial g_t}{\partial \gamma_Q} = 0$ hold.

In the second phase of the industrial era, taking the first derivative of (39) with respect to γ yields

$$\frac{\partial g_t}{\partial \gamma} = \alpha\left[\phi - \frac{2-\theta}{1-\theta}(\mu-1)\left(\frac{1-\gamma}{\mu}\right)^{\frac{1}{1-\theta}} x_t\right] < 0. \tag{D.8}$$

To show that this expression is always negative, let $M(\gamma) = (\mu-1)\left(\frac{1-\gamma}{\mu}\right)^{\frac{1}{1-\theta}} x_t$, so (39) can be written as $g_t(\gamma)+\rho = (1-\gamma)\alpha(M(\gamma)-\phi)$ such that $M(\gamma) = \phi + \frac{g_t+\rho}{(1-\gamma)\alpha}$. On the other hand, the derivative in (D.8) can be written as $\frac{\partial g_t}{\partial \gamma} = \alpha\left[\phi - \frac{2-\theta}{1-\theta}M(\gamma)\right]$. Substituting for $M(\gamma)$ it immediately follows that

$$\frac{\partial g_t}{\partial \gamma} = -\frac{\alpha\phi}{1-\theta} - \frac{2-\theta}{(1-\theta)(1-\gamma)}(g_t+\rho). \tag{D.9}$$

The expression is negative for all admissible parameter values. On the balanced growth path, (D.7) holds. □

Proof of Proposition 4

Proof. First, we set $\gamma_y > 0$ and $\gamma_Q = 0$. The pre-industrial era follows exactly the same dynamic behavior as in the general case. In the first phase of the industrial era, (D.1) and (D.2) hold. In the second phase of the industrial era, (D.3) holds. On the balanced growth path, $\frac{\partial g^*}{\partial \gamma_y} = 0$ holds.

Next, we set $\gamma_y = 0$ and $\gamma_Q > 0$. The pre-industrial era follows exactly the same dynamic behavior as in the general case. By setting $\gamma_y = 0$ in (D.1) and (D.2), it immediately follows that $\frac{\partial x_N}{\partial \gamma_y} = 0$ and $\frac{\partial g_t}{\partial \gamma_y} = 0$. In the second phase of the industrial era, setting $\gamma_y = 0$ in (D.4) yields

$$\frac{\partial g_t}{\partial \gamma_Q} = -\alpha\left[(\mu-1)\left(\frac{1}{\mu}\right)^{\frac{1}{1-\theta}} x_t - \phi\right] < 0. \tag{D.10}$$

On the balanced growth path, (D.6) and (D.7) hold. □

Data availability

Data will be made available on request.

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